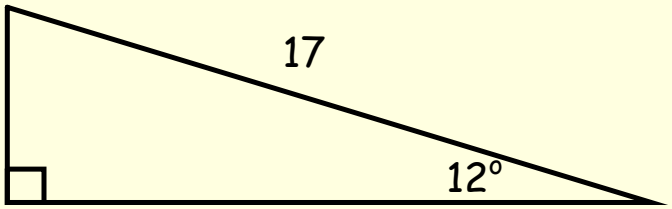
8.1 Law of Sines

Solving Triangles

If you know 3 parts of any triangle,
you can always find the other 3 parts.

Right Triangle: use Trig functions and Pythagorean theorem



But not all triangles are right triangles...
how do we solve the others??

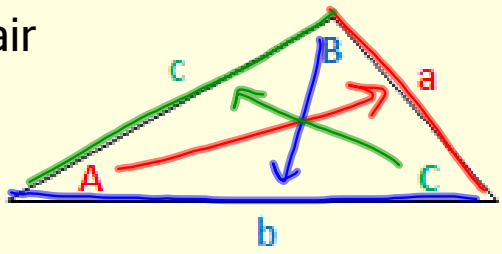
Law of Sines

or

Law of Cosines

The Law of Sines

for use with a non-right triangle
when we know a side-angle pair



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

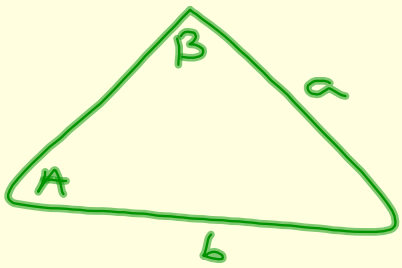
Proof

Use the Law of Sines to completely solve each triangle:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Straightforward cases with ONE solution: AAS or ASA

1. $\angle A = 36°, \angle B = 48°, a = 8$



÷ $\frac{\sin 36}{8} \rightarrow \frac{\sin 48}{b}$

$b = 10.1$

$\angle C = 180 - 36 - 48$

$\angle C = 96°$

$\frac{\sin 36}{8} = \frac{\sin 96}{c}$

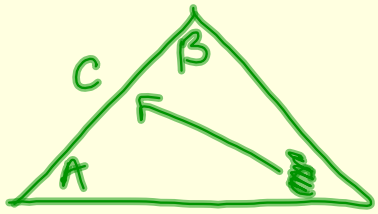
$c = 13.5$

Use the Law of Sines to completely solve each triangle:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Straightforward cases with ONE solution: AAS or ASA

2. $\angle A = 38°, \angle B = 63°, c = 15$



$\angle C = 79°$

$\frac{\sin 79}{15} = \frac{\sin 38}{a}$

$a = 9.41$

$\frac{\sin 79}{15} = \frac{\sin 63}{b}$

$b = 13.6$

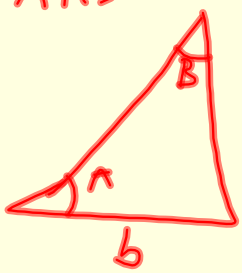
Use the Law of Sines to completely solve each triangle:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Straightforward cases with ONE solution: AAS or ASA

3. $b = 41,\ \angle A = 33°,\ \angle B = 29°$

AAS



$\frac{\sin 29}{41} = \frac{\sin 33}{a}$

$a = 41 \sin 33 / \sin 29 = 46.1$

$\angle C = 180 - 33 - 29 = 118°$

$\frac{\sin 29}{41} = \frac{\sin 118}{c}$

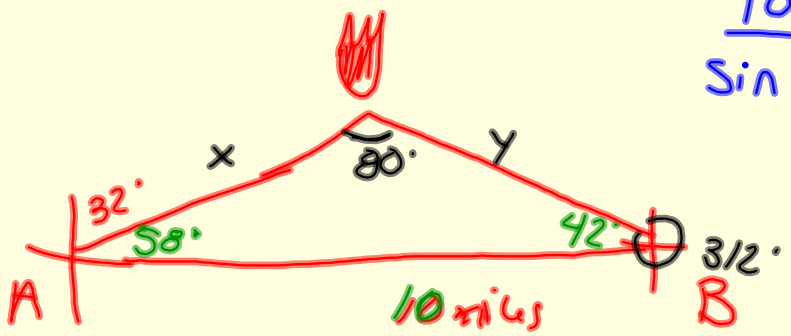
$c = 41 \sin 118 / \sin 29 = 74.7$

Use the Law of Sines to completely solve each triangle:

Straightforward cases with ONE solution: AAS or ASA

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

4. Forest Ranger Chris Johnson at ranger station A sights a fire at a bearing of 032°. Ranger Rick Thorpe at ranger station B, 10 miles due east of A, sights the same fire at a bearing of 312°. Find the distance of each ranger to the fire.



$\frac{10}{\sin 80} = \frac{x}{\sin 42}$

$x = 6.79$ miles (Ranger CJ)

$\frac{\sin 58}{y} = \frac{\sin 80}{10}$

$y = 8.61$ miles (Ranger RT)

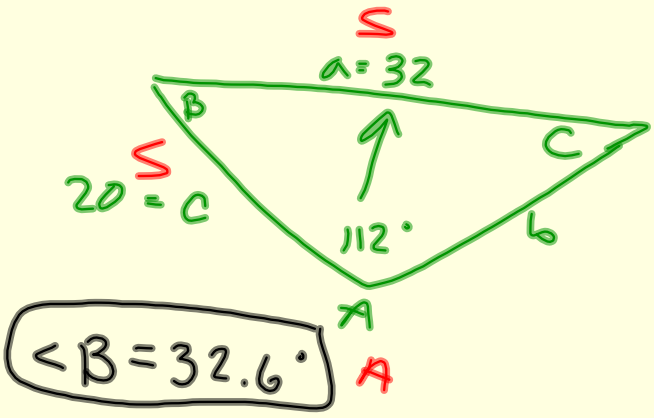
The Law of Sines

the ambiguous case

Side-Side- Angle... presents a problem

The ambiguous case with ONE, ZERO, or TWO solutions: SSA

5. One solution: $a = 32,\ c = 20,\ \angle A = 112°$



$\frac{\sin 112}{32} = \frac{\sin C}{20}$

$32 \sin C = 20 \sin 112$

$\sin C = 0.57948$

$C = \sin^{-1}(0.57948)$

$\angle C = 35.4°$

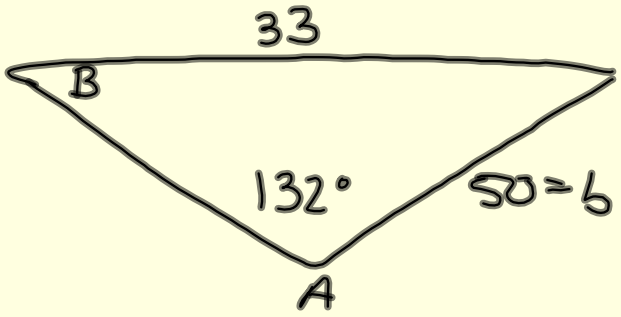
$\angle B = 32.6°$ A

$\frac{\sin 112}{32} = \frac{\sin 32.6}{b}$

$b = 18.6$

The ambiguous case with ONE, ZERO, or TWO solutions: SSA

6. No solutions: $b = 50,\ a = 33,\ \angle A = 132^\circ$



$$\frac{33}{\sin 132} = \frac{50}{\sin B}$$

$$\sin B = 1.13$$

$$\angle B = \varnothing$$

The ambiguous case with ONE, ZERO, or TWO solutions: SSA

7. Two solutions: $a = 125,\ b = 150,\ \angle A = 25°$

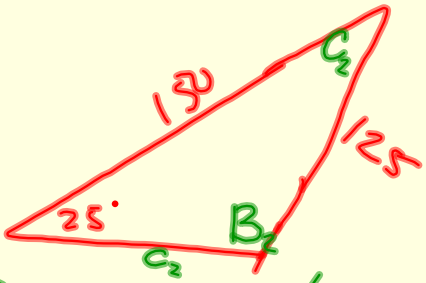
$\frac{\sin 25}{125} = \frac{\sin B}{150}$

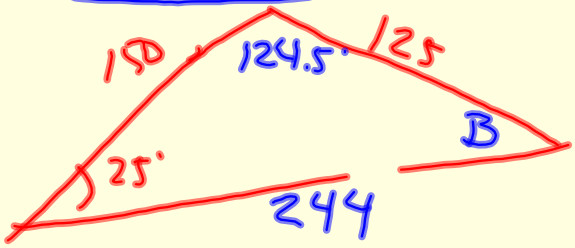
$\angle B_1 = 30.5°$

$\angle C_1 = 124.5°$

$\frac{\sin 25}{125} = \frac{\sin 124.5}{c_1}$

$c_1 = 244$





$\angle B_2 = 149.5°$ $(180 - 30.5)$

$\angle C_2 = 5.5°$

$c_2 = 28.3$

$\frac{\sin 25}{125} = \frac{\sin 5.5}{c_2}$

Homework Assignment

page 517: 7-13 odd, 27-33 odd
[29 and 31 have TWO solutions],
43, 51