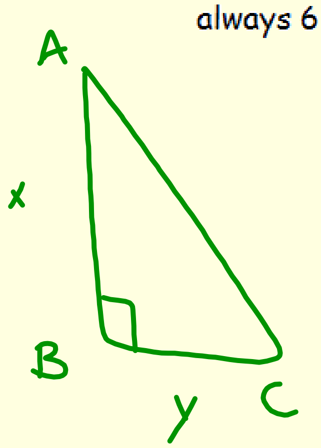
CALCULUS Unit #2 : Applications of Derivatives

7.7a Optimization Problems

Another main application of the derivative is "optimization". This kind of problem asks you to find the maximum or minimum possible value of a length, an area, a volume, a cost, and so on.

The difficulty lies not in taking the derivative, but in finding the equations which accurately represent the situation. There will always be two equations, one of which you will have to substitute into the other.

Example 1: A right triangle ABC has lengths AB and BC such that their sum is always 6 cm. Find the maximum possible area of triangle ABC.



$x + y = 6 \rightarrow y = 6 - x$

$A = \frac{1}{2}xy$

$\boxed{A = \frac{1}{2}x(6-x)}$

$A = 3x - \frac{1}{2}x^2$

$A' = 3 - x$

$0 = 3 - x$

$x = 3$ max

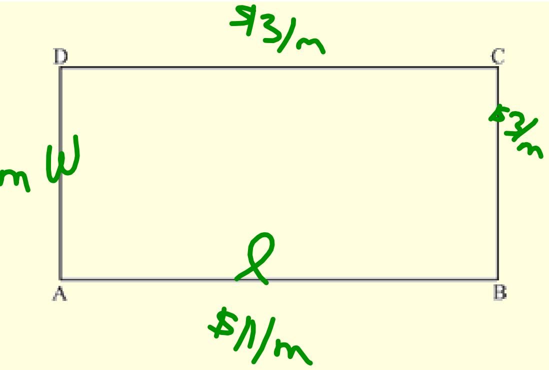
$A'' = -1$ ccv

$\boxed{\begin{matrix} x = 3 \\ y = 3 \end{matrix}}$

$A = \frac{1}{2}(3)(3)$

$9/2\ cm^2$

Example 2: A farmer wishes to create a rectangular enclosure, ABCD, of area 525 m^2, as shown to the right. The fencing used for side AB costs \$11 per meter. The fencing for the other three sides costs \$3 per meter.



The farmer creates an enclosure so that the cost is a minimum. Find this minimum cost.

$$\boxed{525 = lw} \qquad l = \frac{525}{w}$$

$$C = 11l + 3w + 3w + 3l$$

$$C = 6w + 14l$$

$$C = 6w + 14\left(\frac{525}{w}\right)$$

$$C = 6w + 7350w^{-1}$$

$$C' = 6 - 7350w^{-2} = 0$$

$$\frac{6}{1} = \frac{7350}{w^2}$$

$$6w^2 = 7350$$

$$\sqrt{w^2} = \sqrt{1225}$$

$$w = 35$$

$$\boxed{\begin{array}{l} W = 35m \\ l = 15m \end{array}}$$

$$C = \$420$$

$$C'' = \frac{14700}{w^3}$$

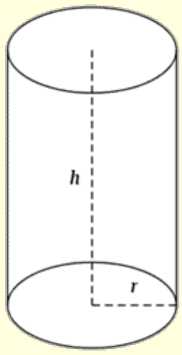
+

CC ↑

minimum

Example 3: A closed, right circular cylinder of base radius *r* cm and height *h* cm has a volume of 54π cm³.

Find the minimum possible surface area.



$54\cancel{\pi} = \cancel{\pi} r^2 h$

$SA = 2\pi r^2 + 2\pi r h$

$\frac{54}{r^2} = \frac{r^2 h}{r^2}$

$h = \frac{54}{r^2}$

$SA = 2\pi r^2 + 2\pi r\left(\frac{54}{r^2}\right)$

$SA = 2\pi r^2 + 2\pi\left(\frac{54}{r}\right)$

$SA = 2\pi r^2 + 108\pi r^{-1}$

$SA' = 4\pi r - 108\pi r^{-2}$

$0 = 4\pi r(1 - 27r^{-3})$

$\cancel{4\pi r = 0}$ $\cancel{r = 0}$

$1 - 27r^{-3} = 0$

$1 = 27r^{-3}$

$\frac{1}{27} = \frac{1}{r^3}$

$r = 3$

$h = \frac{54}{3^2}$

$h = 6$

$SA = 2\pi(3)^2 + \frac{108\pi}{3}$

$SA = 18\pi + 36\pi = 54\pi \text{ cm}^2$

Homework:

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