CALCULUS Unit #2 : Applications of Derivatives

7.6 First and Second Tests, Curve Sketching

A Summary of Curve Sketching

Key feature on curve	How to find it
y-intercept	$x=0$
x-intercept(s)	$y=0$
local min/max	$f'(x)=0$
inflection point(s)	$f''(x)=0$
vertical asymptotes $x=a$	denominator $=0$
horizontal asymptotes $y=a$	

$$\frac{x^3}{x} \quad \frac{x}{x^2} \quad \frac{3x^2}{2x^2}$$

Horizontal asymptote at y = 0

1. $f(x) = \frac{1}{x+3}$

3. $f(x) = \frac{5-2x}{x^2+3x}$

4. $f(x) = \frac{-5}{x^2-4}$

5. $f(x) = \frac{x+1}{x^2-x-12}$

Horizontal asymptote at y = constant

2. $f(x) = \frac{4x-1}{2x-1}$

7. $f(x) = \frac{4x^2-25}{x^2-1}$

8. $f(x) = \frac{x^2+2x-3}{x^2-2x-3}$

9. $f(x) = \frac{2x^2-2}{x^2-4}$

No horizontal asymptote

6. $f(x) = \frac{x^2-x-6}{x-1}$

10. $f(x) = \frac{x^2-5x}{x+5}$

Rules for Horizontal Asymptotes

Degree Relationship	H.A.

6. *[Maximum mark: 5]*

A function f has its first derivative given by $f'(x) = (x-3)^3$.

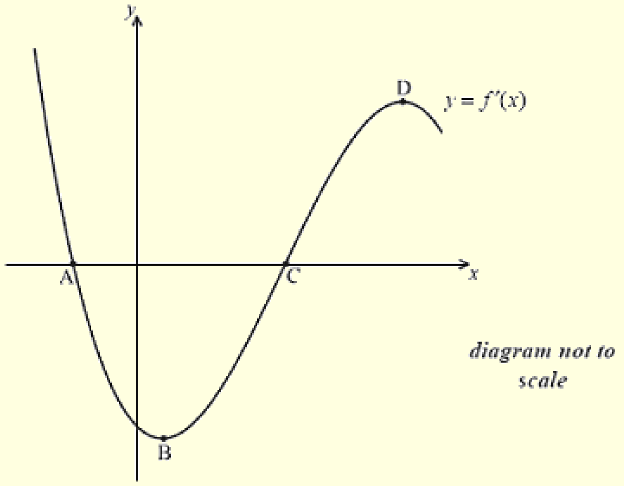
(a) Find the second derivative. *[2 marks]*

(b) Find $f'(3)$ and $f''(3)$. *[1 mark]*

(c) The point P on the graph of f has x-coordinate 3. Explain why P is not a point of inflexion. *[2 marks]*

6. *[Maximum mark: 7]*

The diagram shows part of the graph of $y = f'(x)$. The x-intercepts are at points A and C. There is a minimum at B, and a maximum at D.



(a) (i) Write down the value of $f'(x)$ at C.

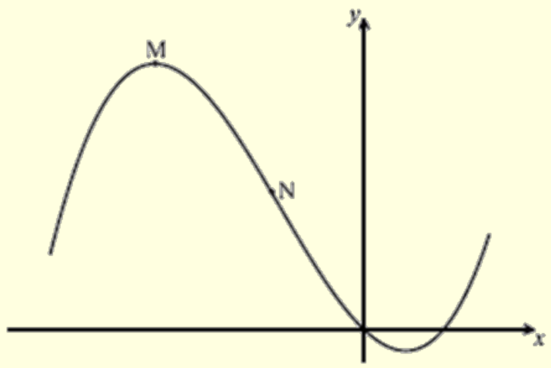
(ii) **Hence**, show that C corresponds to a minimum on the graph of f, *i.e.* it has the same x-coordinate. *[3 marks]*

(b) Which of the points A, B, D corresponds to a maximum on the graph of f? *[1 mark]*

(c) Show that B corresponds to a point of inflexion on the graph of f. *[3 marks]*

8. *[Maximum mark: 14]*

Consider $f(x) = \frac{1}{3}x^3 + 2x^2 - 5x$. Part of the graph of f is shown below. There is a maximum point at M, and a point of inflexion at N.



(a) Find $f'(x)$. *[3 marks]*

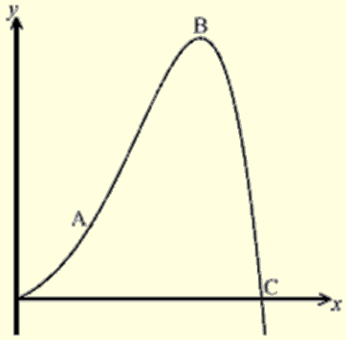
(b) Find the x-coordinate of M. *[4 marks]*

(c) Find the x-coordinate of N. *[3 marks]*

(d) The line L is the tangent to the curve of f at (3, 12). Find the equation of L in the form $y = ax + b$. *[4 marks]*

Part B *[Maximum mark: 15]*

The function f is defined as $f(x) = e^x \sin x$, where x is in radians. Part of the curve of f is shown below.



There is a point of inflexion at A, and a local maximum point at B. The curve of f intersects the x-axis at the point C.

(a) Write down the x-coordinate of the point C. *[1 mark]*

(b) (i) Find $f'(x)$.

(ii) Write down the value of $f'(x)$ at the point B. *[4 marks]*

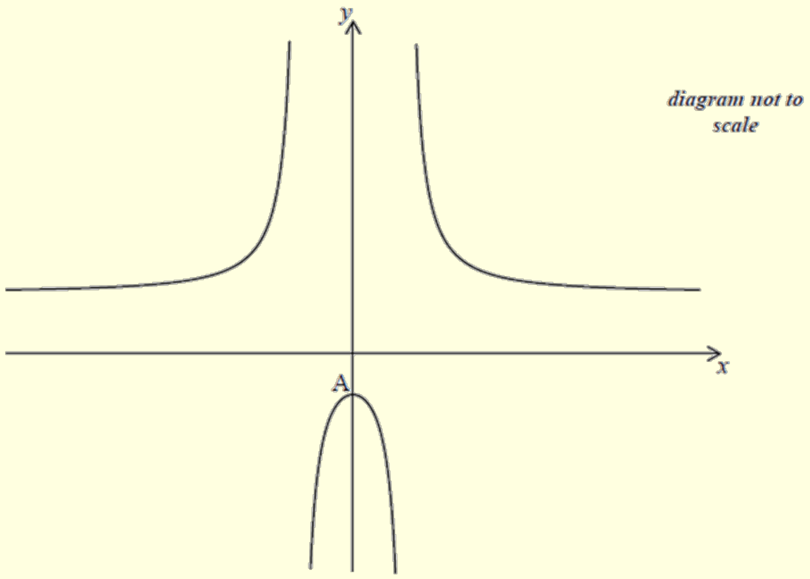
(c) Show that $f''(x) = 2e^x \cos x$. *[2 marks]*

(d) (i) Write down the value of $f''(x)$ at A, the point of inflexion.

(ii) Hence, calculate the coordinates of A. *[4 marks]*

1. *[Maximum mark: 16]*

Let $f(x) = 3 + \frac{20}{x^2 - 4}$, for $x \neq \pm 2$. The graph of f is given below.



The y-intercept is at the point A.

(a) (i) Find the coordinates of A.

(ii) Show that $f'(x) = 0$ at A. *[7 marks]*

(b) The second derivative $f''(x) = \frac{40(3x^2+4)}{(x^2-4)^3}$. Use this to

(i) justify that the graph of f has a local maximum at A:

(ii) explain why the graph of f does **not** have a point of inflexion. *[6 marks]*

(c) Describe the behaviour of the graph of f for large $|x|$. *[1 mark]*

(d) Write down the range of f. *[2 marks]*

Homework:

complete WS problems