

Objective: Solve exponential and logarithmic functions.

Exponential equations are equations in which variable expressions occur as exponents. The result below is useful for solving certain exponential equations.

### KEY CONCEPT

#### Property of Equality for Exponential Equations

**Algebra** If  $b$  is a positive number other than 1, then  $b^x = b^y$  if and only if  $x = y$ .

**Example** If  $3^x = 3^5$ , then  $x = 5$ . If  $x = 5$ , then  $3^x = 3^5$ .

Solve.  $4 = 2^2$      $\frac{1}{2} = 2^{-1}$

1.  $4^x = \left(\frac{1}{2}\right)^{x-3}$

$$2^{2x} = 2^{-1(x-3)}$$

only exponents:  $2x = -x + 3$

$$3x = 3$$

$$x = 1$$

3.  $100^{7x+1} = 1000^{3x-2}$

$$10^{2(7x+1)} = 10^{3(3x-2)}$$

$$14x + 2 = 9x - 6$$

$$5x = -8$$

$$x = -\frac{8}{5}$$

5.  $4^x = 11$

$$\log_4 11 = x$$

$$x = \frac{\log 11}{\log 4}$$

$$x \approx 1.730$$

2.  $9^{2x} = 27^{x-1}$

$$3^{2(2x)} = 3^{3(x-1)}$$

$$2(2x) = 3(x-1)$$

$$4x = 3x - 3$$

$$x = -3$$

4.  $81^{3-x} = \left(\frac{1}{3}\right)^{5x-6}$

$$3^{4(3-x)} = 3^{-1(5x-6)}$$

$$12 - 4x = -5x + 6$$

$$x = -6$$

6.  $2^x = 5$

$$\log_2 5 = x$$

$$x = \frac{\log 5}{\log 2}$$

$$x \approx 2.322$$

$$\log_e = \ln$$

7.  $7^{9x} = 15$

$$\log_7 15 = 9x$$

$$\boxed{x \approx 0.155}$$

8.  $4e^{-0.3x} - 7 = 13$

$$4e^{-0.3x} = 20$$

$$e^{-0.3x} = 5$$

$$\ln 5 = -0.3x$$

$$x = \frac{\ln 5}{-0.3} \approx \boxed{-5.365}$$

9.  $\log_5 (4x - 7) = \log_5 (x + 5)$

$$4x - 7 = x + 5$$

$$3x = 12$$

$$\boxed{x = 4}$$

10.  $\ln (7x - 4) = \ln (2x + 11)$

$$7x - 4 = 2x + 11$$

$$5x = 15$$

$$\boxed{x = 3}$$

11.  $\log_4 (5x - 1) = 3$

$$4^3 = 5x - 1$$

$$64 = 5x - 1$$

$$65 = 5x$$

$$\boxed{x = 13}$$

12.  $\log_2 (x - 6) = 5$

$$2^5 = x - 6$$

$$32 = x - 6$$

$$\boxed{38 = x}$$

13.  $\log 5x + \log (x - 1) = 2$

condense  
 $\log 5x(x-1) = 2$

$$10^2 = 5x(x-1)$$

$$100 = 5x^2 - 5x$$

$$0 = 5x^2 - 5x - 100$$

$$0 = 5(x^2 - x - 20)$$

$$0 = 5(x-5)(x+4)$$

$$\boxed{x = 5, \cancel{x = -4}}$$

14.  $\log_4 (x + 12) + \log_4 x = 3$

condense

$$\log_4 (x+12)(x) = 3$$

$$4^3 = x^2 + 12x$$

$$0 = x^2 + 12x - 64$$

$$0 = (x+16)(x-4)$$

$$x = \cancel{x = -16} \boxed{4}$$