

Objective: Rewrite logarithmic functions.

KEY CONCEPT**Properties of Logarithms**Let b , m , and n be positive numbers such that $b \neq 1$.

Product Property $\log_b mn = \log_b m + \log_b n$

Quotient Property $\log_b \frac{m}{n} = \log_b m - \log_b n$

Power Property $\log_b m^n = n \log_b m$

CHANGE-OF-BASE FORMULA Logarithms with any base other than 10 or e can be written in terms of common or natural logarithms using the *change-of-base formula*. This allows you to evaluate any logarithm using a calculator.

KEY CONCEPT**Change-of-Base Formula**If a , b , and c are positive numbers with $b \neq 1$ and $c \neq 1$, then:

$$\log_c a = \frac{\log_b a}{\log_b c}$$

In particular, $\log_c a = \frac{\log a}{\log c}$ and $\log_c a = \frac{\ln a}{\ln c}$.

Use $\log_4 3 \approx 0.792$ and $\log_4 7 \approx 1.404$

$$\begin{aligned} 1. \quad \log_4 \frac{3}{7} &= \log_4 3 - \log_4 7 \\ &= 0.792 - 1.404 \\ &= -0.612 \end{aligned}$$

$$\begin{aligned} 2. \quad \log_4 21 &= \log_4 7(3) \\ &= \log_4 7 + \log_4 3 \\ &= 1.404 + 0.792 \\ &= 2.196 \end{aligned}$$

$$\begin{aligned} 3. \quad \log_4 49 &= \log_4 7^2 \\ &= 2 \log_4 7 \\ &= 2(1.404) \\ &= 2.808 \end{aligned}$$

$$\begin{aligned} 4. \quad \log_4 9 &= \log_4 3^2 \\ &= 2 \log_4 3 \\ &= 2(0.792) \\ &= 1.584 \end{aligned}$$

Expand.

5. $\log_6 \frac{5x^3}{y}$

$$\begin{aligned} &= \log_6 \underline{5x^3} - \log_6 y \\ &= \log_6 5 + \log_6 x^3 - \log_6 y \\ &= \log_6 5 + 3\log_6 x - \log_6 y \end{aligned}$$

6. $\log 3x^4$

$$\begin{aligned} &= \log 3 + \log x^4 \\ &= \log 3 + 4\log x \end{aligned}$$

Condense

7. $\log 9 + 3 \log 2 - \log 3$

$$\begin{aligned} &= \log 9 + \log 2^3 - \log 3 \\ &= \log 9 \cdot 2^3 - \log 3 \\ &= \log \frac{9 \cdot 2^3}{3} = \log \frac{72}{3} = \log 24 \end{aligned}$$

8. $\ln 4 + 3 \ln 3 - \ln 12$

$$\begin{aligned} &= \ln 4 + \ln 3^3 - \ln 12 \\ &= \ln 4(3^3) - \ln 12 \\ &= \ln \frac{4(3^3)}{12} = \ln \frac{108}{12} \\ &= \ln 9 \end{aligned}$$

Use the change of base formula to evaluate the logarithm.

9. $\log_3 8 = \frac{\log 8}{\log 3}$

$$\approx 1.893$$

10. $\log_5 8 = \frac{\ln 8}{\ln 5}$

$$\approx 1.292$$

11. $\log_8 14 = \frac{\log 14}{\log 8}$

$$\approx 1.269$$

12. $\log_{26} 9 = \frac{\ln 9}{\ln 26}$

$$\approx 0.674$$