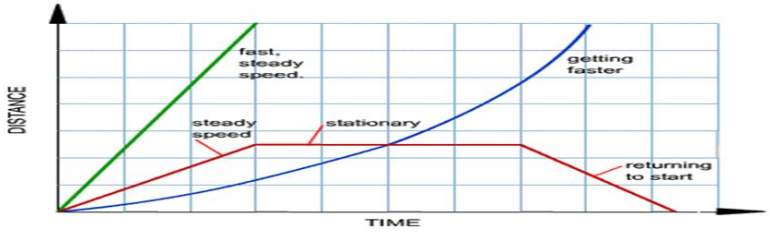
CALCULUS Unit #2 : Applications of Derivatives

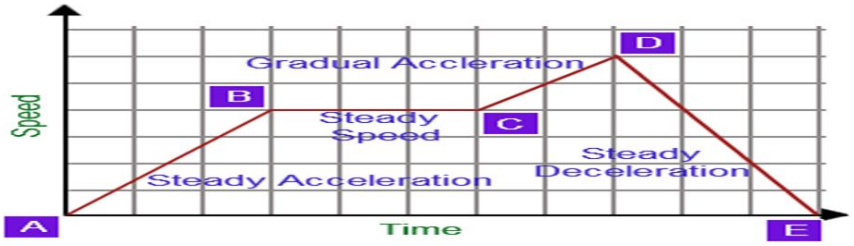
7.5 Kinematics and Rate of Change

Distance/time graphs vs. speed/time graphs

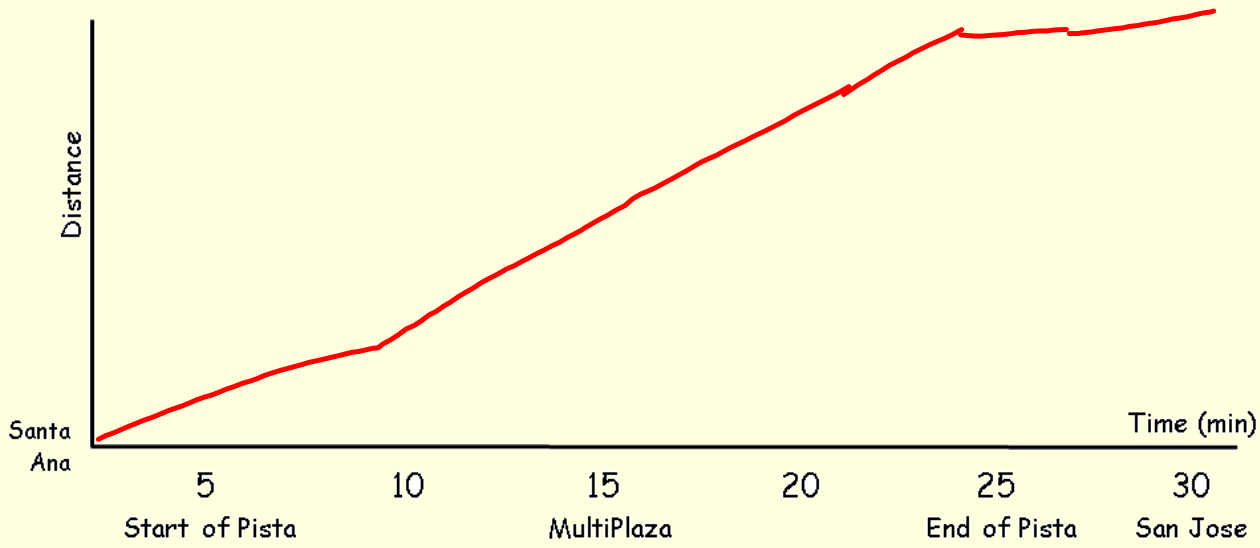
*A **distance-time graph** shows the speed of an object*



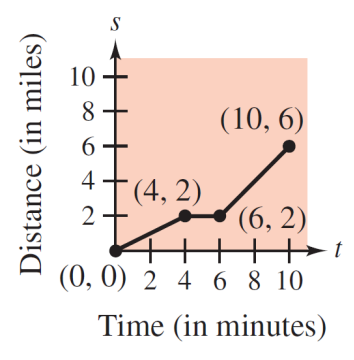
*A **speed-time graph** shows how an object's speed changes over time.*

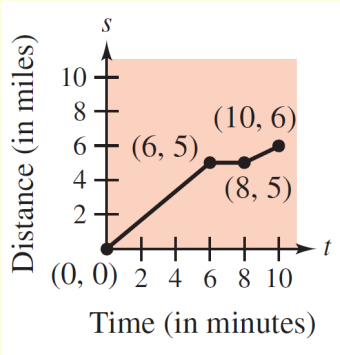


Car Ride from Santa Ana to San Jose



Kinematics


s
Distance (in miles)
10
8
6
4
2
(10, 6)
(4, 2)
(6, 2)
(0, 0) 2 4 6 8 10
t
Time (in minutes)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Distance (in miles)
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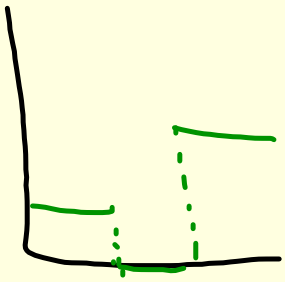
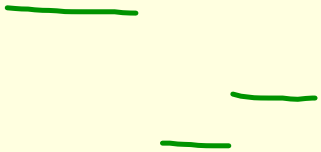
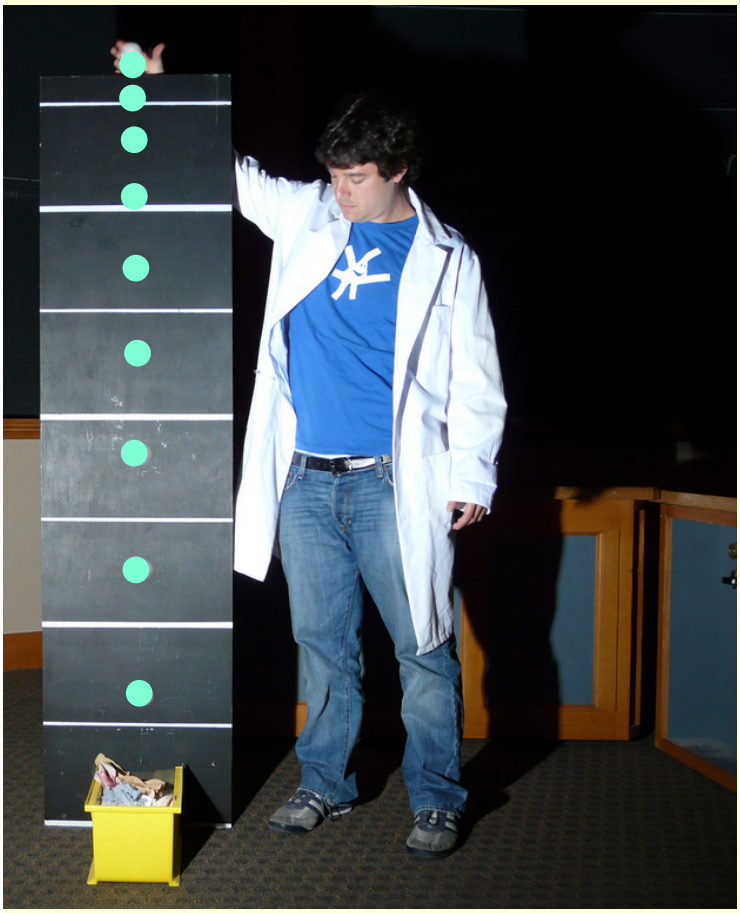
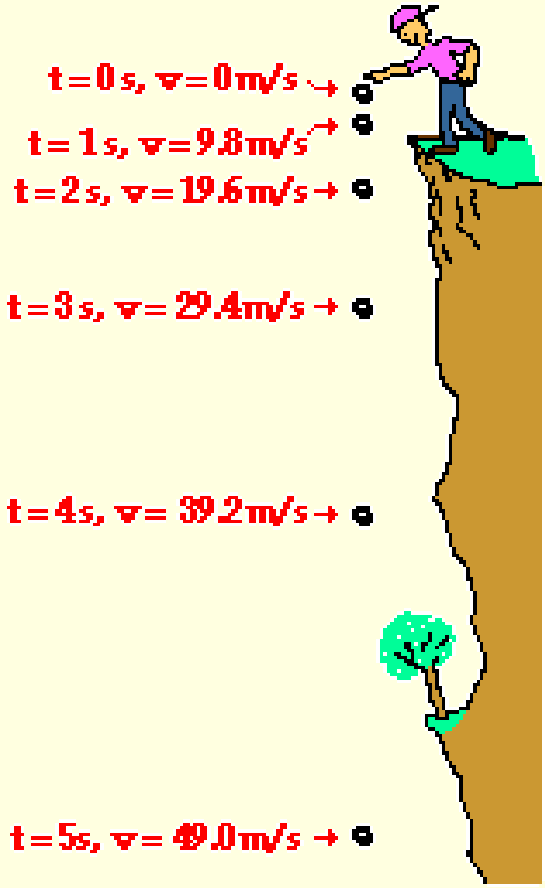
t
Time (in minutes)





Motion with Constant Acceleration





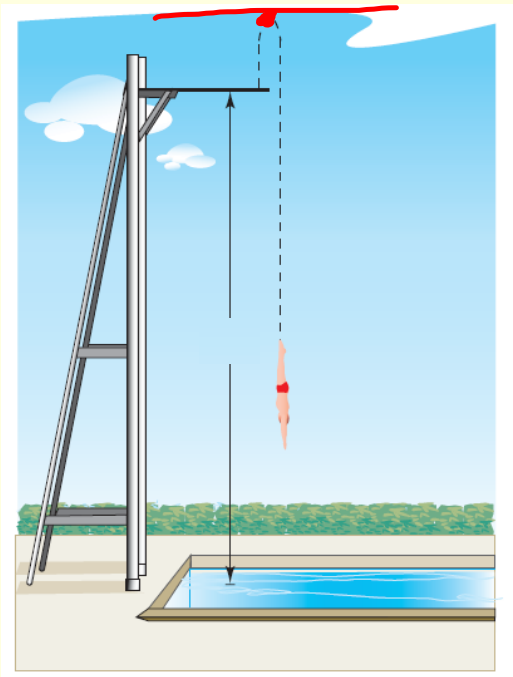
Kinematics

position: $s(t)$

velocity: $v(t) = s'(t)$

acceleration: $a(t) = v'(t) = s''(t)$

Kinematics



Velocity is positive when an object is rising, and is negative when an object is falling.

At time t = 0, a diver jumps from a diving board. The position of the diver is given by

$$s(t) = -4.9t^2 + 4.9t + 9.8$$

a) What is the diver's initial height?
b) What is the diver's initial velocity?
c) When does the diver hit the water?
d) What is the diver's velocity at impact?
e) How high above the board does the diver reach?

Kinematics

3. A particle moves in a straight line with a displacement of s meters t seconds after leaving a fixed point. The displacement function is given by $s(t) = 2t^3 - 21t^2 + 60t + 3$ for $t \geq 0$.

a) Find the velocity of the particle at any time t.
b) Find the initial position and velocity of the particle.
c) Find when the particle is at rest.
d) Find when the particle is moving left and moving right. $\pm v(t)$
e) Find when the particle is speeding up and slowing down. $\pm a(t)$
f) Draw a motion diagram for the particle.

a) $v(t) = 6t^2 - 42t + 60$

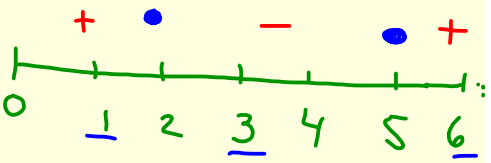
b) $s(0) = 3$, $v(0) = 60$

c) $6t^2 - 42t + 60 = 0$

$6(t^2 - 7t + 10) = 0$

$(t-5)(t-2) = 0$

$t = 5$ sec. and 2 sec.



d) $v(1) = 24$ ⊕ right

$v(3) = -12$ ⊖ left

$v(6) = 24$ ⊕ right

right: $0 < t < 2$ and $t > 5$

left: $2 < t < 5$

e) $a(t) = 12t - 42$

$0 = 12t - 42$

$t = 3.5$ sec.

$\therefore$ $0 < t < 3.5$ slowing

$t > 3.5$ speeding up

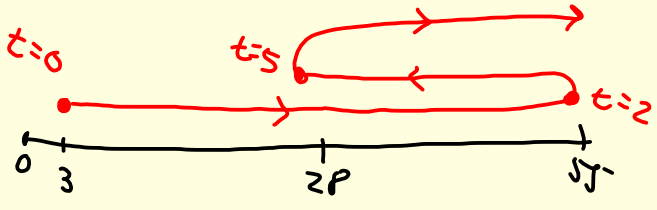
$a(1) = -30$ ⊖ slow

$a(4) = 6$ ⊕ speed

e) $s(0) = 3$ m

$s(2) = 55$ m

$s(5) = 28$ m



Kinematics

4. During one month, the temperature of the water in a pond is modeled by the function $C(t) = 20 + 9te^{-\frac{t}{3}}$, where t is measured in days and C in degrees Celsius.

   a) Find the average rate of change in temperature over the first 15 days of the month.

   b) Find the rate of change in temperature on day 10.

(a) $C(0) = 20$

$C(15) = 20.9$

$\frac{20.9-20}{15-0} = \frac{0.9}{15} = 0.0606 \frac{°C}{day}$

(b) $C'(t) = 9e^{-t/3} + 9te^{-t/3}\left(-\frac{1}{3}\right)$

$C'(10) = 9e^{-10/3} + -3(10)e^{-10/3}$

$= -0.749 \, °C/day$

Homework:

Exercise 7N (1-4)

Exercise 70 (2-3)

Exercise 7P (2-4)