Algebra II: 7.4 Notes
Date:

Name:
Period:

Key

Objective: Evaluate logarithms and graph logarithmic functions.

You know that $2^2 = 4$ and $2^3 = 8$. However, for what value of x does $2^x = 6$? Mathematicians define this x-value using a *logarithm* and write $x = \log_2 6$. The definition of a logarithm can be generalized as follows.

KEY CONCEPT *For Your Notebook*

Definition of Logarithm with Base b

Let b and y be positive numbers with $b \neq 1$. The logarithm of y with base b is denoted by $\log_b y$ and is defined as follows:

$$\log_b y = x \quad \text{if and only if} \quad b^x = y$$

The expression $\log_b y$ is read as "log base b of y."

This definition tells you that the equations $\log_b y = x$ and $b^x = y$ are equivalent. The first is in *logarithmic form* and the second is in *exponential form*.

SPECIAL LOGARITHMS A **common logarithm** is a logarithm with base 10. It is denoted by $\log_{10}$ or simply by log. A **natural logarithm** is a logarithm with base e. It can be denoted by $\log_e$, but is more often denoted by ln.

Common Logarithm	Natural Logarithm
$\log_{10} x = \log x$	$\log_e x = \ln x$

Rewrite the equation in exponential form.

1. $\log_3 81 = 4$ $\quad 3^4 = 81$
2. $\log_7 7 = 1$ $\quad 7^1 = 7$
3. $\log_{14} 1 = 0$ $\quad 14^0 = 1$
4. $\log_{1/2} 32 = -5$ $\quad \left(\frac{1}{2}\right)^{-5} = 32$

Evaluate the logarithm.

5. $\log_4 64$ $\quad 4^x = 64$ $\quad x = 3$
6. $\log_5 0.2$ $\quad 5^x = 0.2$ $\quad 5^x = \frac{1}{5}$ $\quad x = -1$
7. $\log_{1/5} 125$ $\quad \left(\frac{1}{5}\right)^x = 125$ $\quad x = -3$
8. $\log_{36} 6$ $\quad 36^x = 6$ $\quad x = \frac{1}{2}$
9. $\log 8$ $\quad 10^x = 8$ $\quad x = 0.903$
10. $\ln 0.3$ $\quad e^x = 0.3$ $\quad x = -1.204$

INVERSE FUNCTIONS By the definition of a logarithm, it follows that the logarithmic function $g(x) = \log_b x$ is the inverse of the exponential function $f(x) = b^x$. This means that:

$$g(f(x)) = \log_b b^x = x \quad \text{and} \quad f(g(x)) = b^{\log_b x} = x$$

Simplify the expression.

11. $10^{\log 4}$ $= 4$

12. $\log_5 25^x$ $= \log_5 5^{2x} = 2x$

13. $8^{\log_8 x}$ $= x$

14. $\log_7 7^{-3x} = -3x$

15. $\log_2 64^x$ $= \log_2 2^{6x} = 6x$

16. $e^{\ln 20} = 20$

Find the inverse.

17. $y = 6^x$ $x = 6^y$ $y = \log_6 x$

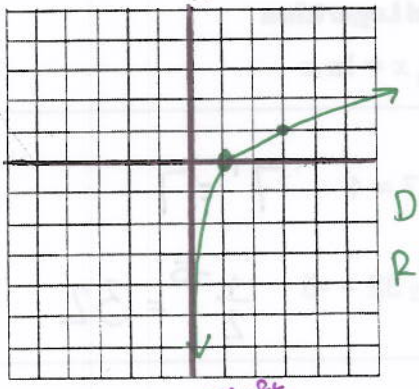
18. $y = \ln(x + 3)$ $x = \ln(y+3)$ $e^x = y+3$ $y = e^x - 3$

19. $y = 4^x$ $x = 4^y$ $y = \log_4 x$

20. $y = \ln(x - 5)$ $x = \ln(y-5)$ $e^x = y-5$ $y = e^x + 5$

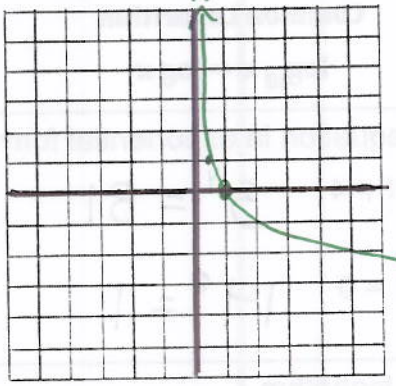
Graph the function, identify the asymptote, and state the domain and range.

21. $y = \log_3 x$



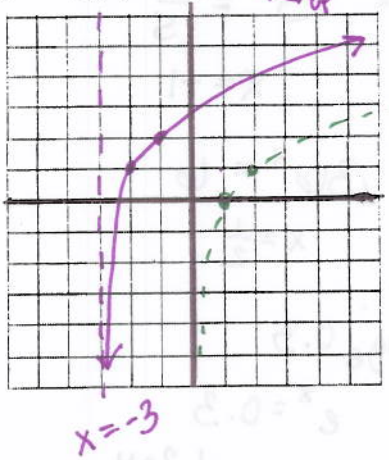
D: $x > 0$
R: $\mathbb{R}$

22. $y = \log_{1/2} x$



D: $x > 0$
R: $\mathbb{R}$

23. $y = \log_2(x + 3) + 1$



D: $x > -3$
R: $y \in \mathbb{R}$