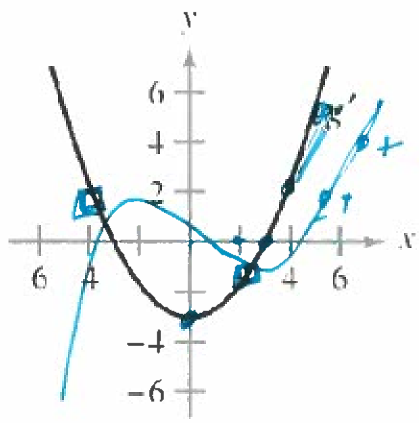
Calculus Unit #1:

7.3 The Product and Quotient Rules and Higher Order Derivatives

64. The figure shows the graph of g'. $\rightarrow x^2$

$g \rightarrow x^3$



(a) $g'(0) =$ [redacted] -3 (b) $g'(3) =$ [redacted] 0

(c) What can you conclude about the graph of g knowing that $g'(1) = -\frac{8}{3}$? $g(x)$ is decreasing @ $x = 1$

(d) What can you conclude about the graph of g knowing that $g'(-4) = \frac{7}{3}$? $g(x)$ is increasing @ $x = -4$

(e) Is $g(6) - g(4)$ positive or negative? Explain. positive

(f) Is it possible to find $g(2)$ from the graph? Explain.

Derivative Rules

1. constant $\to 0$
2. power rule
3. constant multiple
4. $+/-$
5. e^x
6. $\ln x$

What about multiple derivatives???

Find the following second derivatives:

$f(x) = 2x^4 + 3x^3 - x^2 - 4x - 7$

$f'(x) = 8x^3 + 9x^2 - 2x - 4$

$f''(x) = 24x^2 + 18x - 2$

$y = 15e^x$

$y' = 15e^x$

$y'' = 15e^x$

$y = \ln x$

$y' = \frac{1}{x} = x^{-1}$

$y'' = -1x^{-2}$

$= -\frac{1}{x^2}$

As we move into more and more complicated functions, we need additional tools to find derivatives. For example, $f(x) = (3x+1)(x^2-1)$. Maybe we could just multiply the two derivatives together…?

Let $u(x) = 3x + 1$ Then $u'(x) =$ 3

Let $v(x) = x^2 - 1$ Then $v'(x) =$ $2x$

And $u'(x)v'(x) =$ $6x$

But $f(x) =$ $3x^3+x^2-3x-1$ So $f'(x) =$ $9x^2+2x-3$

Examples like this show us that derivatives are much more complicated than they may at first appear. We need additional tools for products and quotients.

Product rule	$y = uv \quad \Rightarrow \quad \dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
Quotient rule	$y = \dfrac{u}{v} \quad \Rightarrow \quad \dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

$y' = uv' + vu'$

$y' = \dfrac{vu' - uv'}{v^2}$

Find the derivative of each function:

1. $h(x) = (3x - 2x^2)(5 + 4x)$ u v
2. $y = \frac{5x-2}{x^2+1}$ $\frac{u}{v}$
3. $y = \frac{9}{5x^2}$ $= \frac{9}{5}x^{-2}$
4. $y = \frac{-3(3x-2x^2)}{7x}$ $= \frac{-9x+6x^2}{7x} = -\frac{9}{7} + \frac{6}{7}x$
5. $f(x) = (3x + 1)(\ln x)$
6. $g(x) = \frac{5x+3}{x^2+1}$
7. $g(x) = \sqrt{x}(4x^2 - 2x)$
8. $f(x) = \frac{x+2}{2e^x-3}$
9. $h(x) = (x^4 + 3x^3 + 6)(2x - 1)$
10. $f(x) = \frac{3x^2+2x+1}{x^2}$
11. $y = \frac{9}{\sqrt[3]{x^4}}$
12. $h(x) = \frac{3x+4}{x^2-2}$

13. Find an equation of the tangent line to the graph of $f(x) = \frac{3-\frac{1}{x}}{x+5}$ at $(-1, 1)$.

① $h'(x) = \underset{u}{(3x-2x^2)}\underset{v'}{(4)} + \underset{v}{(5+4x)}\underset{u'}{(3-4x)}$

$h'(x) = 12x - 8x^2 + 15 - 20x + 12x - 16x^2$

$h'(x) = -24x^2 + 4x + 15$

$y = \frac{u}{v}$

② $y' = \frac{vu' - uv'}{v^2}$

$$y' = \frac{(x^2+1)(5) - (5x-2)(2x)}{(x^2+1)^2} = \frac{5x^2+5-(10x^2-4x)}{(x^2+1)^2}$$

$$= \frac{-5x^2+4x+5}{(x^2+1)^2}$$

③ $y = \frac{9}{5}x^{-2}$

$y' = \frac{-18}{5}x^{-3} = \frac{-18}{5x^3}$

④ $y = \frac{-9}{7} + \frac{6}{7}x$

$y' = \frac{6}{7}$

⑤ $f'(x) = (3x+1)\left(\frac{1}{x}\right) + \ln x\,(3)$

$= 3 + \frac{1}{x} + 3\ln x$

⑥ $g'(x) = \frac{(x^2+1)(5) - (5x+3)(2x)}{(x^2+1)^2}$

$= \frac{5x^2+5-(10x^2+6x)}{(x^2+1)^2} = \frac{-5x^2-6x+5}{(x^2+1)^2}$

⑦ $g(x) = x^{1/2}(4x^2-2x) = 4x^{5/2} - 2x^{3/2}$

$g'(x) = 10x^{3/2} - 3x^{1/2}$

⑧ $f'(x) = \frac{(2e^x-3)(1) - (x+2)(2e^x)}{(2e^x-3)^2} = \frac{2e^x-3-2xe^x-4e^x}{(2e^x-3)^2}$

$= \frac{-2e^x-2xe^x-3}{(2e^x-3)^2}$

⑨ $h'(x) = (x^4+3x^3+6)(2) + (4x^3+9x^2)(2x-1)$

$= 2x^4+6x^3+12 + 8x^4+18x^3-4x^3-9x^2$

$= 10x^4+20x^3-9x^2+12$

(10) $f(x) = \dfrac{3x^2+2x+1}{x^2} = 3 + 2x^{-1} + x^{-2}$

$f'(x) = -2x^{-2} - 2x^{-3} = \dfrac{-2}{x^2} - \dfrac{2}{x^3}$

(11) $y = 9x^{-\frac{4}{3}}$

$y' = \dfrac{-36}{3} x^{-7/3} = -12x^{-7/3} = \dfrac{-12}{x^{7/3}}$

(12) $h'(x) = \dfrac{(x^2-2)(3) - (3x+4)(2x)}{(x^2-2)^2}$

$= \dfrac{3x^2-6-6x^2-8x}{(x^2-2)^2} = \dfrac{-3x^2-8x-6}{(x^2-2)^2}$

Homework:

page 212: 1-10 odd

page 215: 1-18 odd