

Objective: Study functions involving the natural  $e$ .

The history of mathematics is marked by the discovery of special numbers such as  $\pi$  and  $i$ . Another special number is denoted by the letter  $e$ . The number is called the **natural base  $e$**  or the *Euler number* after its discoverer, Leonhard Euler (1707–1783). The expression  $\left(1 + \frac{1}{n}\right)^n$  approaches  $e$  as  $n$  increases.

$n$	$10^1$	$10^2$	$10^3$	$10^4$	$10^5$	$10^6$
$\left(1 + \frac{1}{n}\right)^n$	2.59374	2.70481	2.71692	2.71815	2.71827	2.71828

### KEY CONCEPT

#### The Natural Base $e$

The natural base  $e$  is irrational. It is defined as follows:

As  $n$  approaches  $+\infty$ ,  $\left(1 + \frac{1}{n}\right)^n$  approaches  $e \approx 2.718281828$ .

Simplify the expression.

1.  $e^2 \cdot e^5 = e^7$

2.  $e^7 \cdot e^4 = e^{11}$

3.  $\frac{12e^4}{3e^3} = 4e$

4.  $\frac{24e^8}{4e^5} = 6e^3$

5.  $(5e^{-3x})^2 = 25e^{-6x}$   
 $= \frac{25}{e^{6x}}$

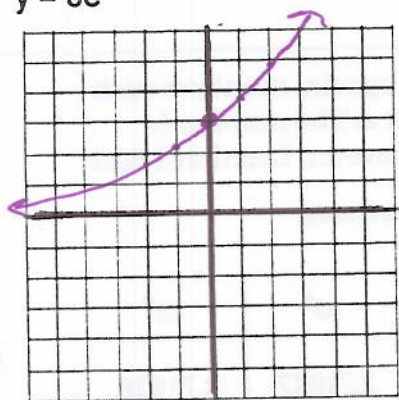
6.  $(10e^{-4x})^3 = \frac{1000}{e^{12x}}$

7.  $2e^{-3} \cdot 6e^5 = 12e^2$

8. Use a calculator to evaluate  $e^{3/4} \approx 2.187$

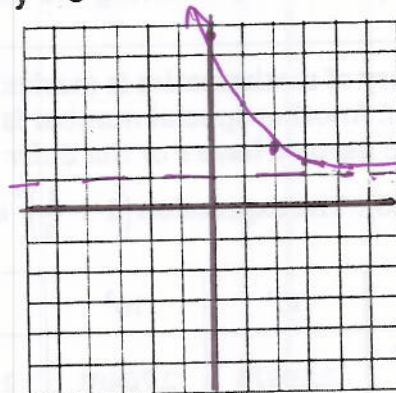
Graph the function, identify the asymptote, and state the domain and range.

9.  $y = 3e^{0.25x}$



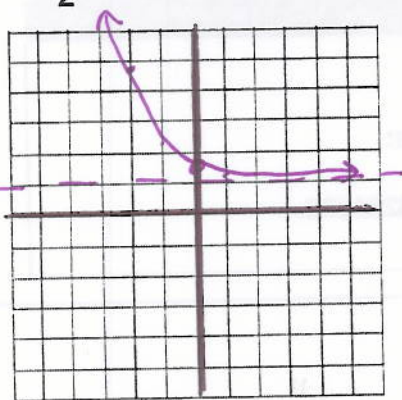
$y = 0$   
 $D: \mathbb{R}$   
 $R: y > 0$

10.  $y = e^{-0.75(x-2)} + 1$



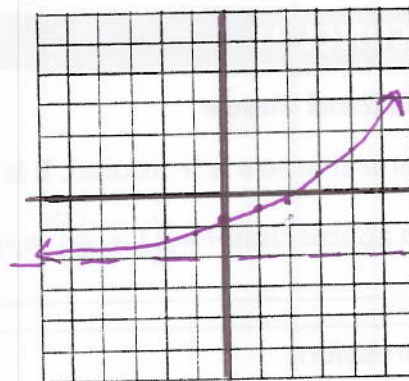
$y = 1$   
 $D: \mathbb{R}$   
 $R: y > 1$

5.  $y = \frac{1}{2}e^{-x} + 1$



$y = 1$   
 $D: \mathbb{R}$   
 $R: y > 1$

6.  $y = 1.5e^{0.25(x-1)} - 2$



$y = -2$   
 $D: \mathbb{R}$   
 $R: y > -2$

### KEY CONCEPT

#### Continuously Compounded Interest

When interest is compounded *continuously*, the amount  $A$  in an account after  $t$  years is given by the formula

$$A = Pe^{rt}$$

where  $P$  is the principal and  $r$  is the annual interest rate expressed as a decimal.

7. You deposit \$4000 in an account that pays 6% annual interest compounded continuously. What is the balance after 1 year?

$$A = 4000e^{0.06(1)}$$

$$A = \$4247.35$$

8. You deposit \$2500 in an account that pays 5% annual interest compounded continuously. What is the balance after 2 years? 7.5 years?

$$A = 2500e^{0.05(2)} = \$2762.93$$

$$A = 2500e^{0.05(7.5)} = \$3637.48$$