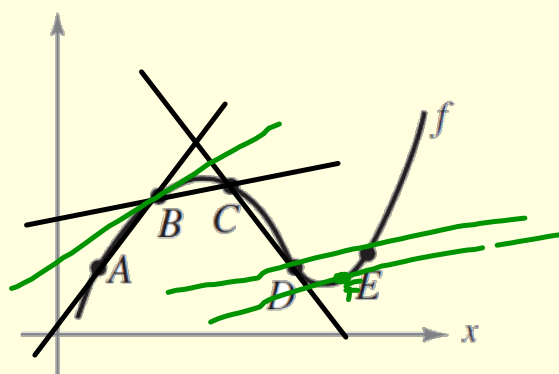


Calculus Unit #1

7.3 The derivative of $\ln x$ and e^x



- a) Between which two consecutive points is the average rate of the change of the function greatest?

A, B or C, D

- b) Is the average rate of change of the function between A and B greater or less than the instantaneous rate of change at B?

A, B

- c) Draw the secant line between D and E. Now, draw a tangent line at point F somewhere between D and E such that the instantaneous rate of change at F is the same as the average rate of change between D and E.

Derivative Rules

$$f(x) = k, \quad f'(x) = 0$$

$$f(x) = \underline{2x} + \underline{7}, \quad \text{sum/diff}$$

constant

$$y = x^3$$

$$\text{Power Rule: } y' = 3x^2$$

$$\frac{1}{3}x^2$$

$$\frac{1}{3}(2x)$$

Log properties
"a log is an exponent"

$$\ln 1 = 0$$

$$\ln e = 1$$

$$\ln(ab) = \ln a + \ln b$$

$$\ln(a/b) = \ln a - \ln b$$

$$\ln(a^n) = n \ln a$$

$$\ln x = a$$

$$e^a = x$$

In Exercises 21–30, use the properties of logarithms to expand the logarithmic expression.

21. $\ln \frac{x}{4}$

23. $\ln \frac{xy}{z}$

25. $\ln(x\sqrt{x^2 + 5})$

27. $\ln \sqrt{\frac{x-1}{x}}$

29. $\ln z(z-1)^2$

22. $\ln \sqrt{x^5} = \ln x^{\frac{5}{2}} = \frac{5}{2} \ln x$

24. $\ln(xyz) = \ln x + \ln y + \ln z$

26. $\ln \sqrt{a-1} = \frac{1}{2} \ln(a-1)$

28. $\ln(3e^2) = \ln 3 + 2$

30. $\ln \frac{1}{e} = \ln 1 - \ln e$
 $= 0 - 1 = -1$

In Exercises 31–36, write the expression as a logarithm of a single quantity.

31. $\ln(x - 2) - \ln(x + 2)$

32. $3 \ln x + 2 \ln y - 4 \ln z$

33. $\frac{1}{3}[2 \ln(x + 3) + \ln x - \ln(x^2 - 1)]$

$= \ln x^3 + \ln y^2 - \ln z^4$

34. $2[\ln x - (\ln(x + 1) + \ln(x - 1))]$

$= \ln \frac{x^3 y^2}{z^4}$

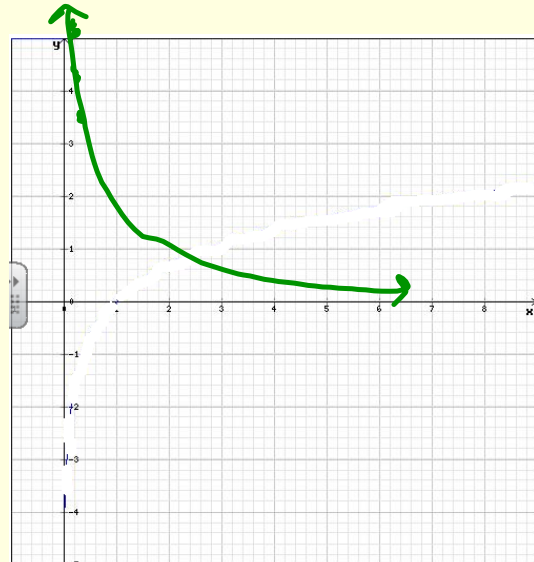
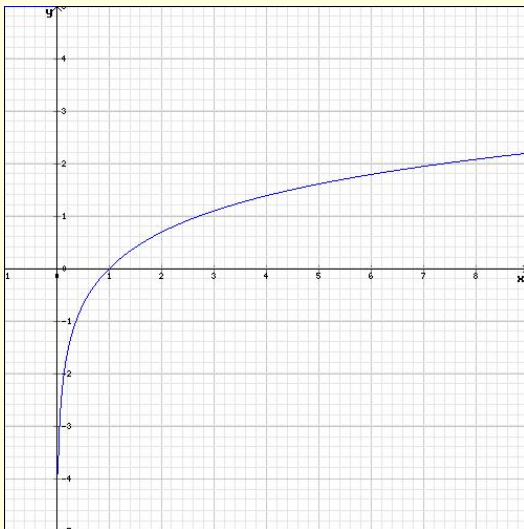
35. $2 \ln 3 - \frac{1}{2} \ln(x^2 + 1)$

36. $\frac{3}{2}[\ln(x^2 + 1) - \ln(x + 1) - \ln(x - 1)]$

34. $2 \left[\ln \left(\frac{x}{(x+1)(x-1)} \right)^2 \right]$

$$\sqrt[b]{a} = \frac{a}{b} \cdot \frac{1}{b}$$

The Derivative of $y = \ln x$

$$\frac{d}{dx}[\ln x] = \frac{1}{x}$$


Exponent properties
"The inverse of logging is
exponenting"

$$e^a e^b = e^{a+b}$$

$$e^a / e^b = e^{a-b}$$

$$(e^a)^b = e^{ab}$$

$$\frac{d}{dx}[\ln x] = \frac{1}{x}$$

$$\frac{d}{dx}[e^x] = e^x$$

Find each derivative:

1) $f(x) = 3e^x$

$$f'(x) = 3e^x$$

2) $f(x) = x^2 + \ln x$

$$f'(x) = 2x + \frac{1}{x}$$

3) $f(x) = \ln e^{3x}$

$$f(x) = 3x$$

$$f'(x) = 3$$

4) Find the equation of the line tangent to the curve $f(x) = 4e^x - 7$ at $x = \ln 3$.

$$f'(x) = 4e^x$$

$$f(\ln 3) = 4e^{\ln 3} - 7$$

$$= 5$$

$$m \rightarrow f'(\ln 3) = 4e^{\ln 3} = 4 \cdot 3 = 12$$

$$m = 12 \quad (\ln 3, 5)$$

$$y - 5 = 12(x - \ln 3)$$

$$3 \ln 3$$

$$3$$

Homework:

page 209-210 (1-6, 8-12all)