From the May 2001 IB exam...

Write the expression $3\sin^2 x + 4\cos x$ in the form $a\cos^2 x + b\cos x + c$.

$= 3(1 - \cos^2 x) + 4\cos x$

$= 3 - 3\cos^2 x + 4\cos x \quad = -3\cos^2 x + 4\cos x + 3$

From a 2000 IB specimen paper...

Write the expression $2\cos^2 x + \sin x$ in terms of $\sin x$ only.

$2(1 - \sin^2 x) + \sin x$

7.3 Double Angle Identities

Intuitive, but Incorrect Formulas

$$(a+b)^2 \neq a^2 + b^2$$

$\sin^2 x =$

$(\sin x)^2$

$\neq \sin x^2$

$$\sqrt{x^2 + y^2} \neq x + y$$

$$\sin 2x \neq 2\sin x$$

$$\cos 2x \neq 2\cos x$$

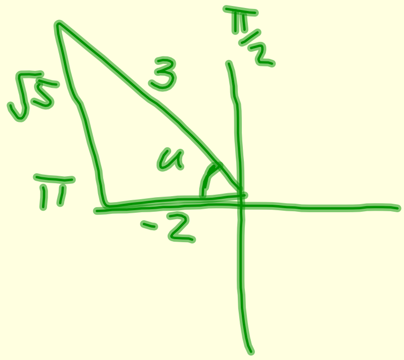
Find $\underset{①}{\sin 2u}$ and $\underset{②}{\cos 2u}$ if $\cos u = \dfrac{-2}{3}\begin{matrix}A\\H\end{matrix}$ and $\dfrac{\pi}{2} < u < \pi$

$$\begin{aligned}\sin 2x &= 2\sin x\cos x\\ \cos 2x &= \cos^2 x - \sin^2 x\\ &= 1-2\sin^2 x\\ &= 2\cos^2 x - 1\end{aligned}$$

① $\sin 2u = 2\sin u\cos u$

$= 2\left(\frac{\sqrt{5}}{3}\right)\left(\frac{-2}{3}\right)$

$= \boxed{\frac{-4\sqrt{5}}{9}}$



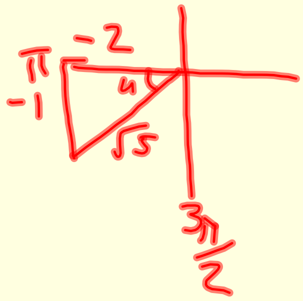
② $\cos 2u = \cos^2 u - \sin^2 u$

$= \left(\frac{-2}{3}\right)^2 - \left(\frac{\sqrt{5}}{3}\right)^2 = \frac{4}{9} - \frac{5}{9} = \boxed{-\frac{1}{9}}$

Find $\sin 2u$ and $\cos 2u$ if $\tan u = \dfrac{1}{2}$ and $\pi < u < \dfrac{3\pi}{2}$

$$\sin 2x = 2\sin x \cos x$$
$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= 1 - 2\sin^2 x$$
$$= 2\cos^2 x - 1$$

$$\sin 2u = 2 \sin u \cos u$$
$$= 2\left(\frac{-1}{\sqrt{5}}\right)\left(\frac{-2}{\sqrt{5}}\right)$$
$$= \boxed{\frac{4}{5}}$$



$$\cos 2u = \cos^2 u - \sin^2 u$$
$$= \left(\frac{-2}{\sqrt{5}}\right)^2 - \left(\frac{-1}{\sqrt{5}}\right)^2$$
$$= \frac{4}{5} - \frac{1}{5} = \boxed{\frac{3}{5}}$$

$$\begin{aligned}\sin 2x &= 2\sin x\cos x\\ \cos 2x &= \cos^2 x - \sin^2 x\\ &= 1-2\sin^2 x\\ &= 2\cos^2 x - 1\end{aligned}$$

Simplify each expression using double angle trigonometric identities:

3. $2\sin\overset{x}{10^\circ}\cos\overset{x}{10^\circ}$

$= \sin 2(10)$

$= \sin 20$

Simplify each expression using double angle trigonometric identities:

$$\begin{aligned} \sin 2x &= 2\sin x\cos x \\ \cos 2x &= \cos^2 x - \sin^2 x \\ &= 1 - 2\sin^2 x \\ &= 2\cos^2 x - 1 \end{aligned}$$

4. $6\sin x\cos x$

$= 3 \cdot 2\sin x\cos x$

$= 3\sin 2x$

Simplify each expression using double angle trigonometric identities:

$$\begin{aligned}\sin 2x &= 2\sin x\cos x\\ \cos 2x &= \cos^2 x - \sin^2 x\\ &= 1 - 2\sin^2 x\\ &= 2\cos^2 x - 1\end{aligned}$$

5. $4 - 8\sin^2 x$

4

$= 4(1 - 2\sin^2 x)$

$= 4\cos 2x$

Simplify each expression using double angle trigonometric identities:

$$\sin 2x = 2\sin x\cos x$$
$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= 1 - 2\sin^2 x$$
$$= 2\cos^2 x - 1$$

6. $$\frac{1+\cos 2x}{\cot x}$$

$$= \frac{\cancel{1} + (2\cos^2 x - \cancel{1})}{\cot x}$$

$$= \frac{2\cos^2 x}{\frac{\cos x}{\sin x}} = 2\cancel{\cos^2}x \cdot \frac{\sin x}{\cancel{\cos x}} = 2\cos x\sin x$$

$$= \boxed{\sin 2x}$$

Simplify each expression using double angle trigonometric identities:

$$\sin 2x = 2\sin x \cos x$$
$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= 1 - 2\sin^2 x$$
$$= 2\cos^2 x - 1$$

7. $$\frac{\sin 2A}{1-\cos 2A} = \frac{2\sin A\cos A}{1+(-1+2\sin^2 A)}$$

$$= \frac{\cancel{2}\,\cancel{\sin A}\cos A}{\cancel{2}\sin^{\cancel{2}} A}$$

$$= \frac{\cos A}{\sin A} = \boxed{\cot A}$$

$\sin 2x = 2\sin x\cos x$
$\cos 2x = \cos^2 x - \sin^2 x$
$= 1 - 2\sin^2 x$
$= 2\cos^2 x - 1$

Prove each identity using double angle trigonometric identities:

8. $\cos^2 A + \cos 2A = 2 - 3\sin^2 A$

$$(1 - \sin^2 A) + (1 - 2\sin^2 A)$$

$$= 2 - 3\sin^2 A$$

$$\begin{aligned} \sin 2x &= 2\sin x \cos x \\ \cos 2x &= \cos^2 x - \sin^2 x \\ &= 1 - 2\sin^2 x \\ &= 2\cos^2 x - 1 \end{aligned}$$

Prove each identity using double angle trigonometric identities:

9. $$\frac{\sin 2x}{1+\cos 2x} = \tan x$$

$$= \frac{\cancel{2}\sin x \cancel{\cos x}}{\cancel{1} + \cancel{2}\cos^2 x \cancel{-1}} = \frac{\sin x}{\cos x} = \tan x$$

$\sin 2x = 2\sin x\cos x$

$1 + \cos 2x = \cos^2 x - \sin^2 x$

$= 1 - 2\sin^2 x$

$= 2\cos^2 x - 1$

Prove each identity using double angle trigonometric identities:

10. $$\frac{\sin x + \sin 2x}{1 + \cos x + \cos 2x} = \tan A$$

$$\frac{\sin x + 2\sin x\cos x}{}$$

$$\begin{aligned} \sin 2x &= 2\sin x\cos x \\ \cos 2x &= \cos^2 x - \sin^2 x \\ &= 1 - 2\sin^2 x \\ &= 2\cos^2 x - 1 \end{aligned}$$

Prove each identity using double angle trigonometric identities:

11. $$1 + \cos 2A = \frac{2}{1 + \tan^2 A}$$

Homework Assignment:

WS 7.3 (back of classwork)