7.2b Geometic Series
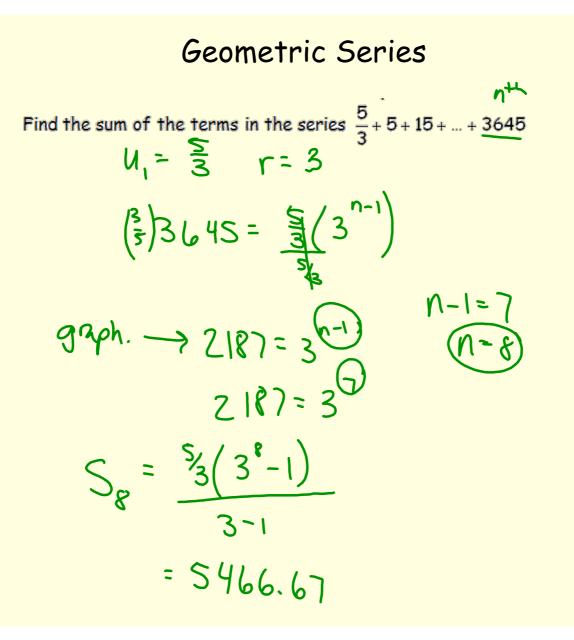
Geometric Series

A <u>geometric series</u> is the <u>sum of terms</u> in a geometric sequence.

The general formula for the sum of the first n terms of a geometric sequence is

$$S_n = \frac{u_1(r^n - 1)}{r - 1} \quad \text{or} \quad S_n = \frac{u_1(1 - r^n)}{1 - r}$$

Geometric Series

Find the sum of the first 8 terms in the sequence 3 - 6 + 12 - …

$$u_1 = 3 \qquad r = -2 \qquad n = 8$$

$$S_8 = \frac{3\left((-2)^8 - 1\right)}{(-2 - 1)}$$

$$= -255$$

Geometric Series

Find the sum of the terms in the series $\frac{5}{3} + 5 + 15 + \ldots + 3645$ ← n^{th}

$$u_1 = \frac{5}{3} \qquad r = 3$$

$$\left(\frac{3}{5}\right) 3645 = \frac{5}{3}\left(3^{n-1}\right)$$

graph. $\longrightarrow$ $2187 = 3^{(n-1)}$

$$2187 = 3^{\textcircled{7}}$$

$$n - 1 = 7$$
$$\boxed{n = 8}$$

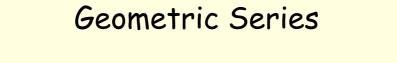$$S_8 = \frac{\frac{5}{3}\left(3^8 - 1\right)}{3 - 1}$$

$$= 5466.67$$

Geometric Series

1. Find the sum of the first ten terms of 12 + 6 + 3 + …

$$u_1 = 12 \qquad r = \tfrac{1}{2}$$

$$S_{10} = \frac{12\left(\tfrac{1}{2}^{10} - 1\right)}{\left(\tfrac{1}{2} - 1\right)}$$

$$= 23.97$$

Geometric Series

2. An employee of a company starts on a salary of $20,000 per year with an annual increase of 4% of the previous year's salary

a) Show that the amounts of the salary form a geometric sequence

$u_1 = 20000$
$u_2 = 20800$ $r = 1.04$
$u_3 = 21632$

b) Find how much the employee earns in the tenth year of employment.

$$u_{10} = 20000 \left(1.04^{10-1}\right) = \$28466.24$$

c) Find the total amount the employee makes over all 10 years.

$$S_{10} = \frac{20000\left(1.04^{10} - 1\right)}{1.04 - 1}$$

$$= \$240,122.14$$

Geometric Series

3. A National Lottery is offering prizes in a new competition. The winner may choose one of the following.

Option one $1000 each week for 10 weeks.

Option two $250 in the first week, $450 in the second week, $650 in the third week, increasing by $200 each week for a total of 10 weeks.

Option three $10 in the first week, $20 in the second week, $40 in the third week continuing to double for a total of 10 weeks.

  a) Calculate the amount you receive in the tenth week for each option.

  b) Calculate the total amount you receive over all ten weeks for each option.

Homework:

page 307-308: 1, 3, 4, 5;
page 308-309:1, 2, 4, 6, 8