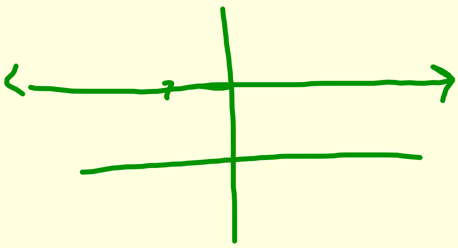
Calculus #1: Basic Differentiation

7.2 Basic Differentiation Rules

Deriving a constant

Function	*Derivative*
a. $y = 7$	$y' = 0$
b. $f(x) = 0$	$f'(x) = 0$
c. $s(t) = -3$	$s'(t) = 0$
d. $y = k\pi^2$, k is constant	$\frac{dy}{dx} = 0$



Deriving a line

Deriving any polynomial

$\frac{1}{3} - \frac{3}{3}$

Function	*Derivative*
a. $f(x) = x^3$	$f'(x) = 3x^2$
b. $g(x) = \sqrt[3]{x} = x^{1/3}$	$g'(x) = \frac{1}{3}x^{-2/3}$
c. $y = \dfrac{1}{x^2} = x^{-2}$	$y' = -2x^{-3} = \dfrac{-2}{x^3}$

Function		*Derivative*
a. $y = \frac{2}{x}$	$= 2x^{-1}$	$y' = -2x^{-2}$
b. $f(t) = \frac{4t^2}{5}$	$= \frac{4}{5}t^2$	$f'(t) = \frac{8}{5}t$
c. $y = 2\sqrt{x}$	$= 2x^{1/2}$	$y' = x^{-1/2}$
d. $y = \frac{1}{2\sqrt[3]{x^2}}$	$= \frac{1}{2}x^{-2/3}$	$y' = -\frac{1}{3}x^{-5/3}$
e. $y = -\frac{3x}{2}$	$= -\frac{3}{2}x$	$y' = -\frac{3}{2}$

Original Function	*Rewrite*	*Differentiate*	*Simplify*
a. $y = \dfrac{5}{2x^3}$			
b. $y = \dfrac{5}{(2x)^3}$			
c. $y = \dfrac{7}{3x^{-2}}$			
d. $y = \dfrac{7}{(3x)^{-2}}$			

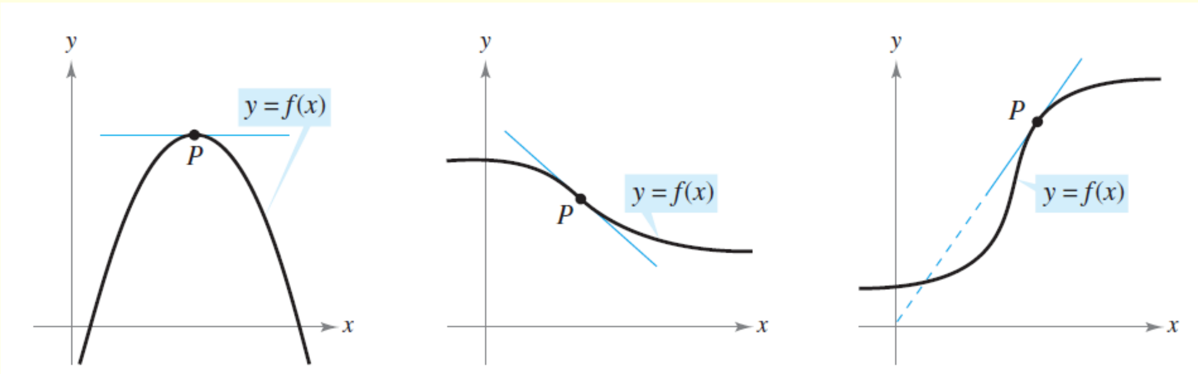
Check for Understanding

Function	*Derivative*
a. $f(x) = x^3 - 4x + 5$	$f'(x) = 3x^2 - 4$
b. $g(x) = -\frac{x^4}{2} + 3x^3 - 2x$	

$g'(x) = -2x^3 + 9x^2 - 2$

$-\frac{1}{2}x^4$

Finding the Equation of the Tangent Line



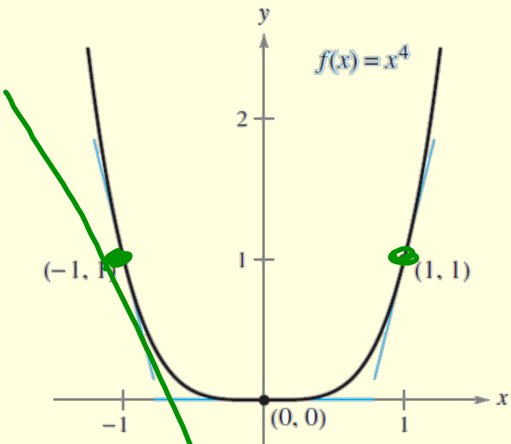
We need:

1) Slope

2) point

Find the slope of the graph of $f(x) = x^4$ when

a. $x = -1$ **b.** $x = 0$ **c.** $x = 1$.



Note that the slope of the graph is negative at the point $(-1, 1)$, the slope is zero at the point $(0, 0)$, and the slope is positive at the point $(1, 1)$.

$f'(x) = 4x^3$

a. $m \to f'(-1) = 4(-1)^3 = -4 = m$

$(-1, 1)$

pt: $x = -1$ $f(-1) = (-1)^4 = 1$

$y - y_1 = m(x - x_1)$

$y - 1 = -4(x + 1)$

$y = -4x - 3$

Find an equation of the tangent line to the graph of $f(x) = x^2$ when $x = -2$.

① $m \rightarrow f'(x) = 2x$

$f'(-2) = 2(-2) = \boxed{-4 = m}$

② ~~pt~~: $f(-2) = (-2)^2 = 4$

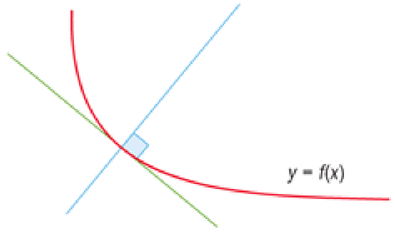
$(-2, 4)$

$y - 4 = -4(x+2)$

$y - 4 = -4x - 8$

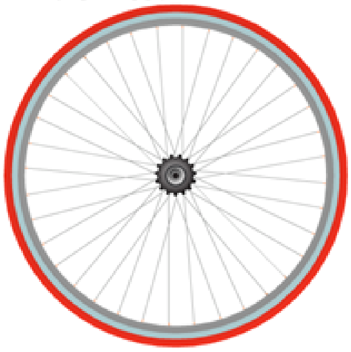
$y = -4x - 4$

The **normal line** at a point on a curve is the line perpendicular to the tangent line at that point.



The slope of the normal line is the opposite reciprocal of the slope of the tangent line

▲ Sparks created by a grinding wheel are **tangent** to the wheel.



▲ Spokes on a bicycle wheel are **normal** to the rim.

Find the equation of the normal line to the curve $f(x) = 2\sqrt{x}$ when $x = 9$.

$2x^{+1/2}$

$f'(x) = x^{-1/2}$

$f'(\cancel{x}9) = 9^{-1/2} = \frac{1}{\sqrt{9}} = \frac{1}{3}$

Slope of normal = -3

pt: $f(9) = 2(\sqrt{9}) = 6$

$(9, 6)$

Find the tangent and normal lines to the curve $f(x) = x + \frac{27}{2x^2}$ when $x = 3$.

Homework:

page 205:1-15 all;

page 207:1, 2ab, 3bc, 5