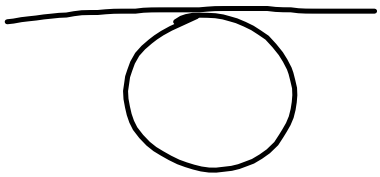
Calculus Unit #1: Basic Differentiation

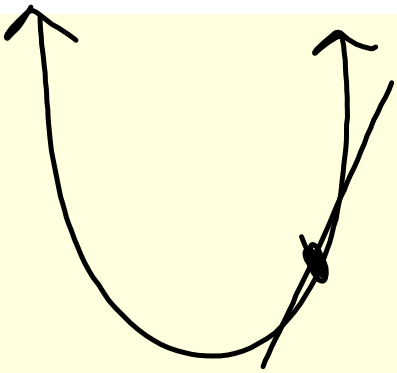
7.2 Average and Instantaneous Rates of Change

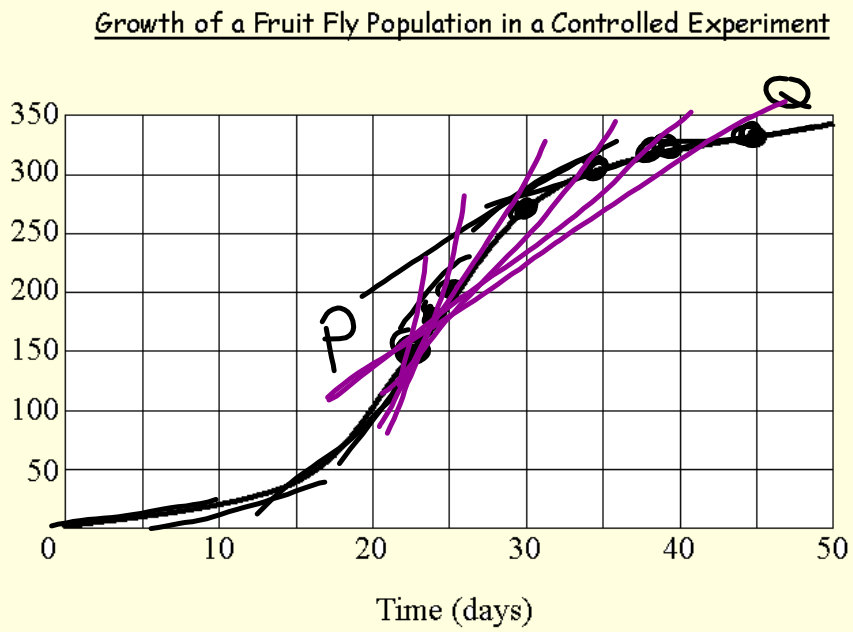
The Tangent Line Problem

Calculus grew out of four major problems that European mathematicians were working on during the seventeenth century.

1. The tangent line problem
2. The velocity and acceleration problem
3. The minimum and maximum problem
4. The area problem

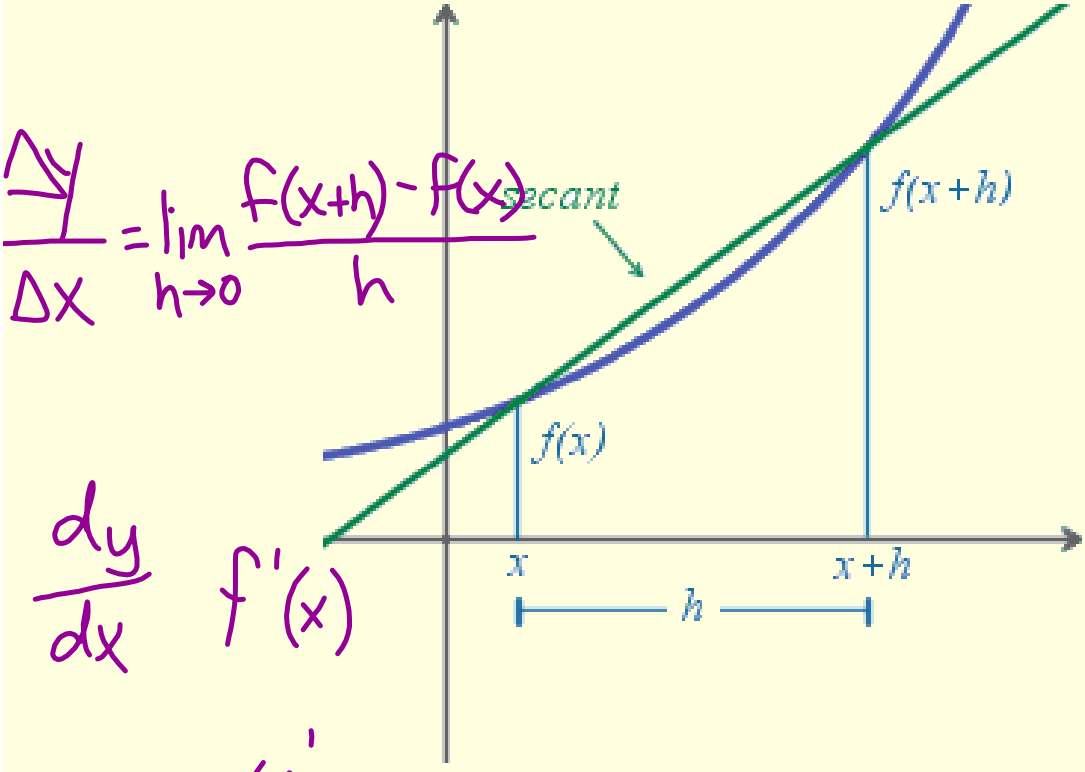






In effect, this is what we're trying to do...

$$\frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$



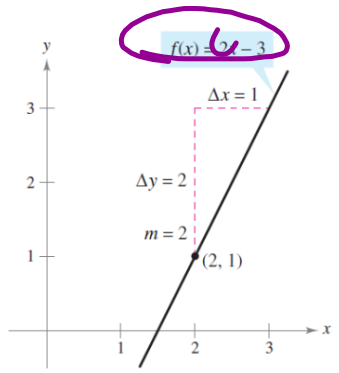
$$\frac{dy}{dx} \quad f'(x)$$

$$y'$$

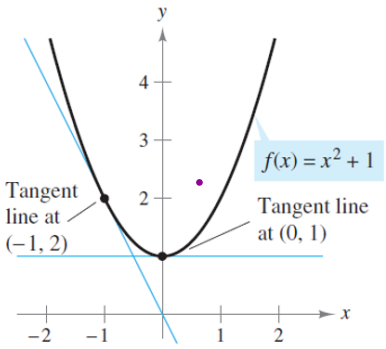
This limiting process on the slope of the secant line gives us the derivative. The derivative of a function gives us the slope of the tangent line at any point x.

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Example 1



Example 2



$$\lim_{h \to 0} \frac{2(x+h) - 3 - (2x-3)}{h}$$

$$\lim_{h \to 0} \frac{2x + 2h - 3 - 2x + 3}{h} = \lim_{h \to 0} \frac{2h}{h} = \lim_{h \to 0} 2 = 2$$

Example 3: Find the derivative of $f(x) = x^3 + 2x$.

$$\lim_{h \to 0} \frac{(x+h)^3 + 2(x+h) - (x^3 + 2x)}{h}$$

1
1 1
1 2 1
1 3 3 1 ←

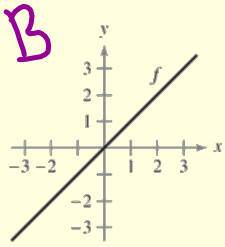
$$\lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 + 2x + 2h - x^3 - 2x}{h}$$

$$\lim_{h \to 0} 3x^2 + 3xh + h^2 + 2$$

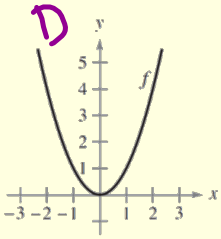
$$= 3x^2 + 2$$

Match each function to its derivative function:

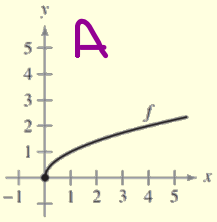
39. B



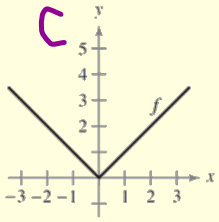
40. D



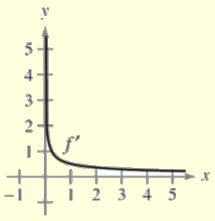
41. A



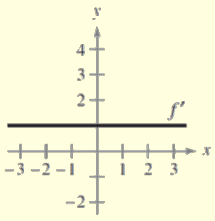
42. C



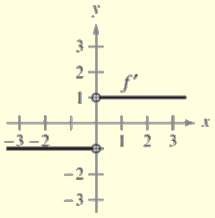
(a)



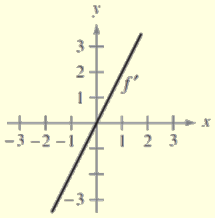
(b)



(c)



(d)



Homework:

Exercises 7C and 7D (p.201)