7.1b Proving Trigonometric Identities

Proofs:

- always start with either the left side or the right side, then show how to get to the other

- usually you start with the more complicated side

- do one step at a time, in a logical order

- each step must be true

- there is more than one way to arrive at the final proof step

- the shortest route is the most "elegant"

<u>Preliminary Example:</u>

Show how LOVE can be changed into MATH....

LOVE $\longrightarrow$ MOVE

MOVE $\longrightarrow$ MOTE   • start at the beginning
                                      • change one letter at a time

MOTE $\longrightarrow$ MOTH   • each middle step is a true word
                                        • there is a beginning and end

MOTH $\longrightarrow$ MATH

Example 1:

Prove　$\tan x + \cot x = \sec x \csc x$

$$\left(\frac{\sin x}{\sin x}\right)\frac{\sin x}{\cos x} \;+\; \frac{\cos x}{\sin x}\left(\frac{\cos x}{\cos x}\right) = \sec x \csc x$$

$$\frac{\sin^2 x}{\sin x \cos x} \;+\; \frac{\cos^2 x}{\sin x \cos x} = \sec x \csc x$$

$$\frac{\sin^2 x + \cos^2 x}{\sin x \cos x} = \frac{1}{\sin x \cos x} = \boxed{\sec x \csc x} \checkmark$$

Example 2:

Prove $\dfrac{\cos t}{1 - \sin t} = \dfrac{1 + \sin t}{\cos t}$

$$\frac{\cos t}{1 - \sin t}\left(\frac{(1 + \sin t)}{(1 + \sin t)}\right)$$

$$= \frac{\cos t(1 + \sin t)}{1 - \sin^2 t}$$

$$= \frac{\cos t(1 + \sin t)}{\cos^2 t} = \boxed{\frac{1 + \sin t}{\cos t}} \checkmark$$

Work in pairs to prove the four examples on your WS:

$$\frac{\sec^2 \theta - 1}{\sec^2 \theta} = \sin^2 \theta$$

$$(\tan^2 x + 1)(\cos^2 x - 1) = -\tan^2 x$$

$$\frac{1}{1 - \sin \alpha} + \frac{1}{1 + \sin \alpha} = 2\sec^2 \alpha$$

$$\tan^4 x = \tan^2 x \sec^2 x - \tan^2 x$$

Homework Assignment:
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