

7.1b Arithmetic Series

Arithmetic Series

A free-falling skydiver speeds up as she falls. That means that each second, she falls a longer distance than the previous second. She falls 16 feet during the first second, 48 feet during the next second, 80 feet during the third second, and so on.

How far will she fall during the 4th second?
During the tenth second?

$$4^{\text{th}} \text{ sec} = 80 + 32 = 112 \text{ ft}$$

$$10^{\text{th}} \text{ sec} = 16 + (10-1)(32) = 304 \text{ ft.}$$

How far will she fall during all ten seconds?

$$1600 \text{ ft.}$$



Arithmetic Series

There are two formulas that can be used to find the sum of an arithmetic series:

$$S_n = \frac{n}{2}(u_1 + u_n)$$

$$S_n = \frac{n}{2}(2u_1 + (n-1)d)$$

$$S_{10} = \frac{10}{2}(16 + 304)$$
$$= 1600 \text{ ft.}$$

$$S_{10} = \frac{10}{2}(2(16) + (10-1)(32))$$
$$= 1600 \text{ ft.}$$

Arithmetic Series

Find the sum the arithmetic sequence 1, 4, 7, 10, 13, 16.

$$\begin{aligned} S_6 &= \frac{6}{2}(1+16) \\ &= 58 \end{aligned}$$

Arithmetic Series

1. Find the sum of the first 13 terms in the sequence $-5, 1, 7, \dots$

$$S_{13} = \frac{13}{2} \left(2(-5) + (13-1)(6) \right)$$
$$= 403$$

Arithmetic Series

$$d = 6$$

2. Find the sum of the terms in the series $-14 - 8 - 2 + \dots + 142$

$$u_n = u_1 + (n-1)(d) \quad \left(S_n = \frac{n}{2} (-14 + 142) \right)$$

Find n:

$$142 = -14 + (n-1)(6)$$

$$156 = 6n - 6$$

$$162 = 6n$$

$$n = 27$$

$$S_{27} = \frac{27}{2} (-14 + 142)$$
$$= \boxed{1728}$$

Arithmetic Series

3. Ron agrees with his son Brett's request for an increase in his allowance of \$0.75 per week for 24 weeks. Brett's allowance now is \$3.00.
- How much will his allowance be on the 24th week?
 - What will be the total amount of allowance that Brett has received over 24 weeks?

$$\begin{aligned} a. \quad u_{24} &= 3 + (24-1)(0.75) \\ &= \$20.25 \end{aligned}$$

$$\begin{aligned} b. \quad S_{24} &= \frac{24}{2}(3 + 20.25) \\ &= \$279 \end{aligned}$$

Check for Understanding #1:

$$S_n = \frac{n}{2} (2u_1 + (n-1)d)$$

$$S_{40} = \frac{40}{2} (2(100) + (40-1)(-7))$$
$$= -1460$$

Check for Understanding #2:

$$S_n = \frac{n}{2} (u_1 + u_n)$$

Find n:

$$S_n = \frac{n}{2} (35000 + 68250)$$

$$68250 = 35000 + (n-1)(1750)$$

$$33250 = 1750n - 1750$$

$$35000 = 1750n$$

$$n = 20$$

$$S_{20} = \frac{20}{2} (35000 + 68250)$$

$$= 1032,500$$

Homework:

page 301-302: 1, 3, 5, 7;
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