7.1 Fundamental Trigonometric Identities

Remember...

$$\csc x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x} \qquad \cot x = \frac{1}{\tan x} \qquad \cot x = \frac{\cos x}{\sin x}$$

$$\sin x = \frac{1}{\csc x} \qquad \cos x = \frac{1}{\sec x} \qquad \tan x = \frac{1}{\cot x} \qquad \tan x = \frac{\sin x}{\cos x}$$

1. $\dfrac{\cot A}{\tan A} = \dfrac{\frac{1}{\tan A}}{\tan A} = \dfrac{1}{\tan A} \cdot \dfrac{1}{\tan A} = \dfrac{1}{\tan^2 A}$ $(\tan A)^2$

$= \cot^2 A$

2. $\dfrac{\tan z}{\sin z} = \dfrac{\frac{\sin z}{\cos z}}{\sin z} = \dfrac{\sin z}{\cos z} \cdot \dfrac{1}{\sin z} = \dfrac{1}{\cos z}$

$= \sec z$

3. $\cos y \csc y = \cos y \cdot \frac{1}{\sin y} = \frac{\cos y}{\sin y} = \cot y$

4. $\tan A \csc A$

$= \frac{\sin A}{\cos A} \cdot \frac{1}{\sin A} = \frac{1}{\cos A} = \sec A$

5. $\dfrac{\sin^2\beta\cot\beta}{\cos\beta} = \dfrac{\sin\beta\,\sin\beta\,\cot\beta}{\cos\beta}$

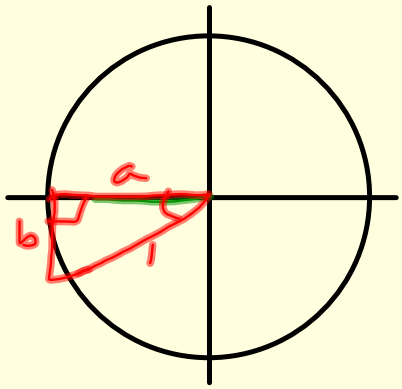
$= \dfrac{\sin\beta\,\sin\beta\,\dfrac{\cos\beta}{\sin\beta}}{\cos\beta} = \sin\beta$

6. $\tan A\cos^2 A$

$= \dfrac{\sin A}{\cos A}\cdot\cos A\cos A$

$= \sin A\cos A$

Pythagorean Identities...



$a^2 + b^2 = 1$

$\sin x = \frac{b}{1}$

$\cos x = \frac{a}{1}$

$\sin^2 x + \cos^2 x = 1$

$1 + \cot^2 x = \csc^2 x$

$\tan^2 x + 1 = \sec^2 x$

WS for Practice

$\sin^2\theta - 1 = -\cos^2\theta$

$$\sin^2 x + \cos^2 x = 1$$
$$1 + \cot^2 x = \csc^2 x$$
$$\tan^2 x + 1 = \sec^2 x$$

7. $\dfrac{\sin^2 x + \cos^2 x}{\cos^2 x} = \dfrac{1}{\cos^2 x} = \sec^2 x$

8. $\sin^2\theta\cos^2\theta - \cos^2\theta$

$= \cos^2\theta(\sin^2\theta - 1)$

$= \cos^2\theta(-\cos^2\theta)$

$= -\cos^4\theta$

$$\sin^2 x + \cos^2 x = 1$$
$$1 + \cot^2 x = \csc^2 x$$
$$\tan^2 x + 1 = \sec^2 x$$

9. $\csc^2\theta - \cot^2\theta$

$(1 + \cot^2\theta) - \cot^2\theta = 1$

10. $\dfrac{1}{\sin^2\theta} - \dfrac{\cos^2\theta}{\sin^2\theta}$

$= \csc^2\theta - \cot^2\theta$

$= 1$

OR

$= \dfrac{1}{\sin^2\theta} - \dfrac{1-\sin^2\theta}{\sin^2\theta}$

$= \dfrac{1 + (-1 + \sin^2\theta)}{\sin^2\theta}$

$= \dfrac{\sin^2\theta}{\sin^2\theta} = 1$

$$\sin^2 x + \cos^2 x = 1$$
$$1 + \cot^2 x = \csc^2 x$$
$$\tan^2 x + 1 = \sec^2 x$$

11. $(1 - \sin x)(1 + \sin x)$ $= 1 - \sin^2 x$
$= \cos^2 x$

12. $\dfrac{\cos^2 A}{1 + \sin A}$

$= \dfrac{1 - \sin^2 A}{1 + \sin A}$ $= \dfrac{(1 - \sin A)(1 + \sin A)}{1 + \sin A}$
$= 1 - \sin A$

$$\sin^2 x + \cos^2 x = 1$$
$$1 + \cot^2 x = \csc^2 x$$
$$\tan^2 x + 1 = \sec^2 x$$

13. $\dfrac{\cos\theta}{\sec\theta - \tan\theta}$

$= \dfrac{\cos\theta}{\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}} = \dfrac{\cos\theta}{\frac{1-\sin\theta}{\cos\theta}}$

$= \cos\theta \cdot \dfrac{\cos\theta}{1-\sin\theta} = \dfrac{1-\sin^2\theta}{1-\sin\theta}$

$= \dfrac{(1+\sin\theta)(1-\sin\theta)}{1-\sin\theta} = 1+\sin\theta$

Strategies:

- use reciprocal identities
- use pythagorean identities
- change everything to sine or cosine
- find common denominators
- use conjugates
- Factor

Homework Assignment:
Worksheet 7.1 HW