Discrete Math: 6.2 Complete Graphs
Date:

Name: Key
Period:

Complete Graph: a graph with N vertices in which every pair of distinct vertices is joined by an edge

Number of Edges in K_N:

$$\frac{N(N-1)}{2}$$

Number of Hamilton Circuits in K_N:

$$(N-1)!$$

Evaluate.

1. Given 9! = 362,880, find 8!.

$9! = 9\cdot8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1 = 362,880$

$8! = 8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1 = \frac{362,880}{9} = 40320$

2. Find 15!/13!. $\frac{15\cdot14\cdot13\cdot12\cdot\ldots}{13\cdot12\cdot11\cdot\ldots} = 15\cdot14 = 210$

Find the value of each of the following.

3. (19! + 21!)/20!

$\frac{19!}{20!} + \frac{21!}{20!} = \frac{1}{20} + \frac{21}{1} = 21.05$

4. (101! + 99!)/100!

$\frac{101!}{100!} + \frac{99!}{100!} = 101 + \frac{1}{100} = 101.01$

Compute.

5. 4! $4\cdot3\cdot2\cdot1 = 24$

6. 6! $6\cdot5\cdot4\cdot3\cdot2\cdot1 = 720$

7. The number of Hamilton circuits in K_5.

$(5-1)! = 4! = 24$

Suppose you have a supercomputer that can generate one billion circuits per second.

8. Estimate in years how long it would take the supercomputer to generate all the Hamilton circuits in K_{20}. $(20-1)! = 6,402,373,705,728,000$

$\div 1,000,000,000 = 6,402,373.7$ sec

$\div 86400$ sec/day $= 74.1$ days

$= 0.2$ years ($\frac{1}{5}$ of a yr.)

9. Estimate in years how long it would take the supercomputer to generate all the Hamilton circuits in K_{21}. $(21-1)! = 2432902008176640000$

$\div 1,000,000,000 = 2432902008$ sec.

$\div 86400 = 28158.6$ days

$\div 365 = 77.1$ years

10. How many edges are there in K_6? $\frac{6(6-1)}{2} = 15$

11. How many edges are there in K_8? $\frac{8(8-1)}{2} = 28$

12. If the number of edges in K_{50} is x, and the number of edges in K_{51} is y, what is the value of y – x ? $\frac{51(50)}{2} - \frac{50(49)}{2} = 25(51-49) = 50$

In each case, find the value of N.

13. K_N has 2 distinct Hamilton circuits.

$N = 3$

14. K_N has 28 edges.

$\frac{N(N-1)}{2} = 28$ $N(N-1) = 56$

$N = 8$

15. K_N has 16,653 edges.

$\frac{N(N-1)}{2} = 16,653$

$N(N-1) = 33306$

$N = 183$

Ch 6: 18-28 evens