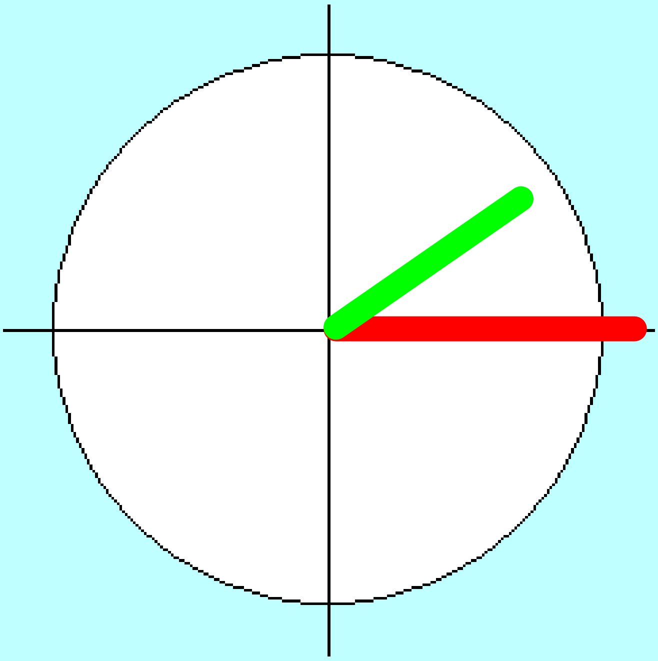
TRIGONOMETRY!

(aren't you excited...!?)

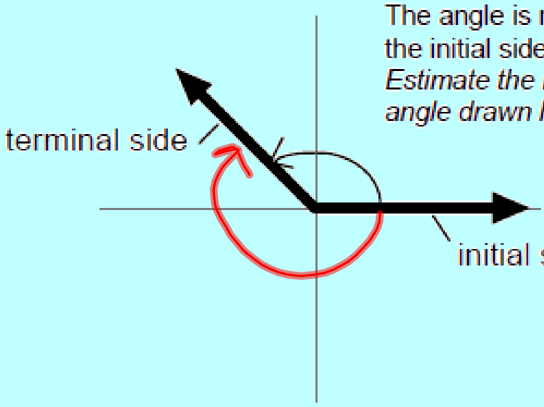
Circle Trigonometry



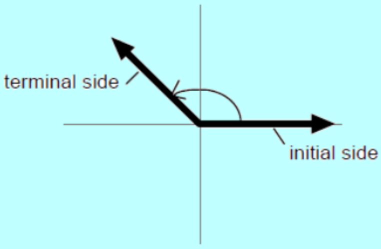
Angles in Standard Position

When we draw an angle in standard position, the vertex of the angle is centered on the origin of a coordinate plane, and the *initial side* of the angle (the first ray) is along the positive x-axis. The *terminal side* of the angle (the other ray) is drawn by measuring the angle counterclockwise from the initial side. If the angle is negative, then it is measured clockwise from the initial side.

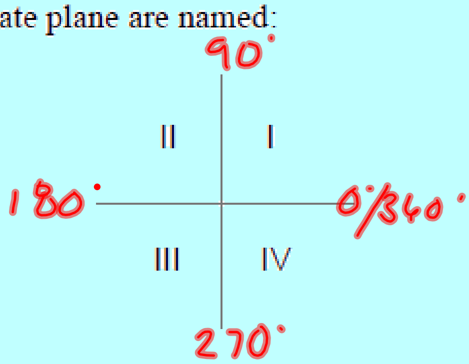
For example:



The angle is measured ccw from the initial side to the terminal side.
Estimate the measurement of the angle drawn here: 120/135 °



The four quadrants of a coordinate plane are named:



In which quadrant does the terminal side of the angle drawn previously lie? II

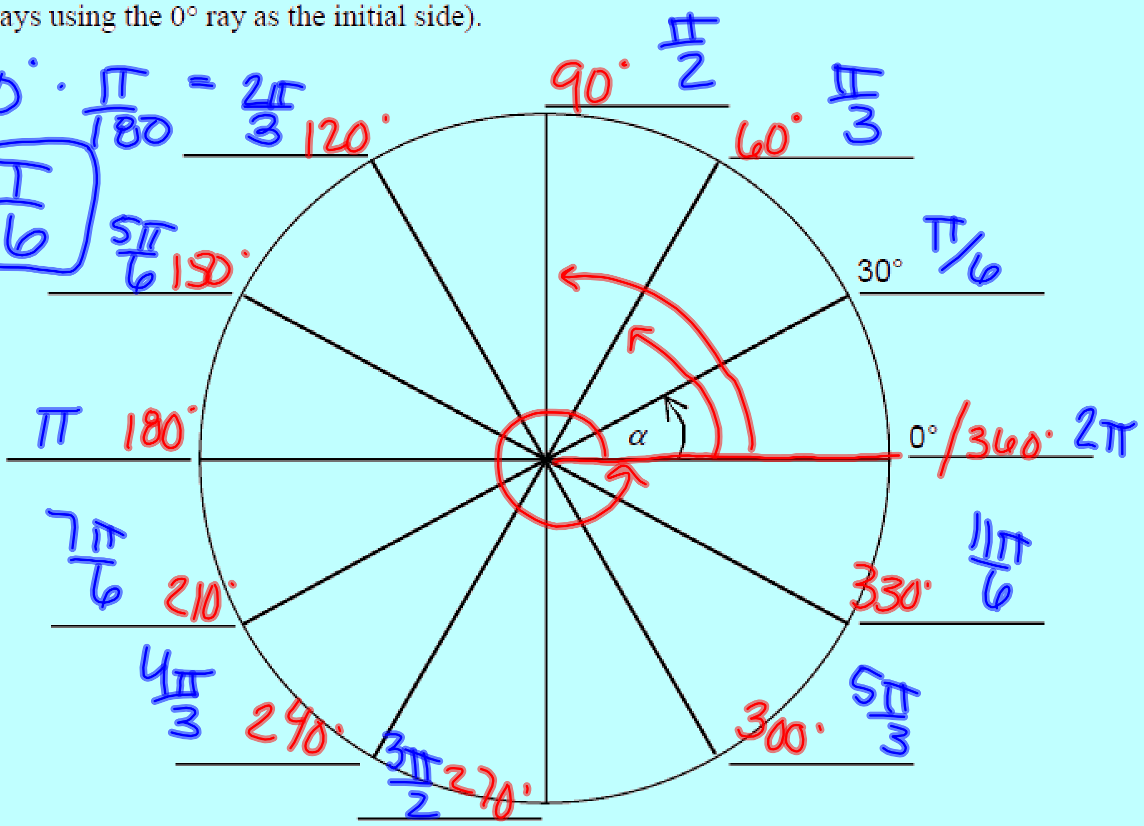
In which quadrant does the terminal side of a 310° angle lie? IV

In which quadrant does the terminal side of a -100° angle lie? III

In which quadrant does the terminal side of a 180° angle lie? II/III

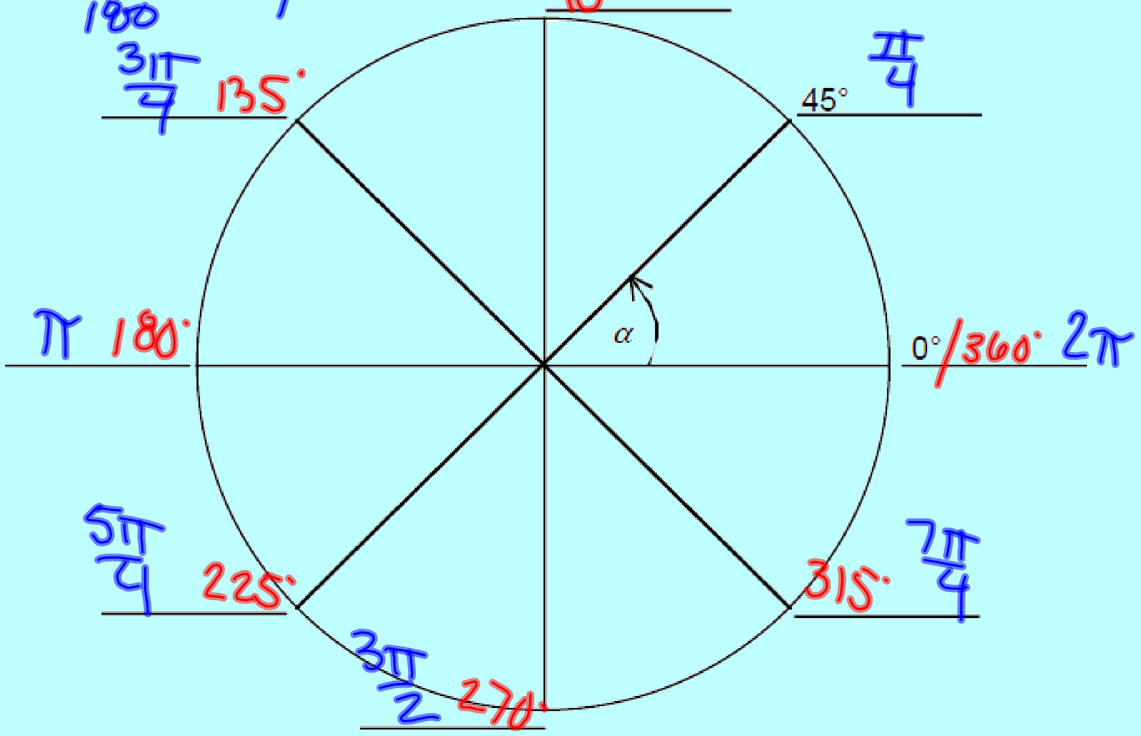
The circle below is marked into 12 equal angles. In degrees, every **sector** has a **central angle** of 30°. *On each ray, write the measure of the angle in* *standard position* *that* *terminates* *at that ray.* (always using the 0° ray as the initial side).

$$30^\circ \cdot \frac{\pi}{180} = \boxed{\frac{\pi}{6}}$$



The circle below is marked into 8 equal angles. In degrees, every **sector** has a **central angle** of 45°. *On each ray, write the measure of the angle in* <u>*standard position*</u> *that* <u>*terminates*</u> *at that ray.* (always using the 0° ray as the initial side).

$45° \cdot \frac{\pi}{180} = \frac{\pi}{4}$



Degree and Radian Measure

We measure angles in 2 different units: degrees and radians.

One circle = 360° (degrees)
One circle = 2π radians

π radians = 180°

To convert an angle measurement from **degrees to radians**, multiply the angle by the ratio, $\boxed{\frac{\pi}{180°}}$. If the angle measured in degrees is θ, then the angle in radians is $\theta \cdot \frac{\pi}{180}$.

Example:

$$\theta = 200°$$

θ

$$\theta \text{ (in radians)} = 200° \cdot \frac{\pi}{180°} = \boxed{\frac{10\pi}{9}}$$

To convert an angle measurement from **radians to degrees**, multiply the angle by the ratio, $\frac{180°}{\pi}$. If the angle measured in radians is α, then the angle in degrees is $\alpha \cdot \frac{180}{\pi}$.

Example:

$$\alpha = \frac{3\pi}{5} \text{ radians}$$

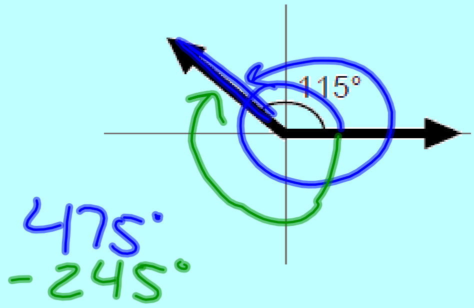
$$\alpha \text{ (in degrees)} = \frac{3\cancel{\pi}}{5} \cdot \frac{180°}{\cancel{\pi}} = 108°$$

Now, we need to look back at the same 2 circles and fill in the radian measures alongside the degree measures.

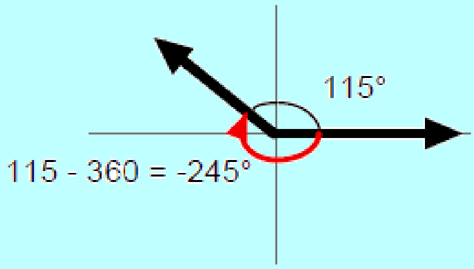
<u>Coterminal Angles</u>

Angles are *coterminal* if they share the same terminal side. Remember, all angles in standard position have the initial side on the positive x-axis. Positive angles open counterclockwise from the initial side to the terminal side, negative angles open clockwise from the initial side to the terminal side.

Can you think of another angle whose terminal side will be the same as the 115° angle shown to the right?



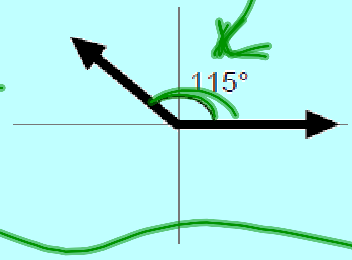
Try starting at the initial side and going <u>clockwise</u> (this is a negative angle).
What negative angle ends at the same terminal side as 115°?



$$115^\circ\left(\frac{\pi}{180}\right) = \frac{23\pi}{36}$$

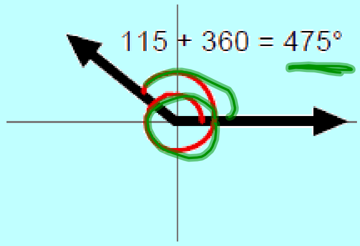
Can you think of another angle whose terminal side will be the same as the 115° angle shown to the right?

$+$



$$\frac{2}{1}\,\frac{\times 36}{\times 36} = \frac{72}{36} \qquad \frac{2\pi}{1} = \frac{72\pi}{36}$$

Try starting at the initial side and going <u>counterclockwise</u> one full circle and then continue until you land on the terminal side again.
What positive angle goes past 360° and then ends at the same terminal side as 115°?



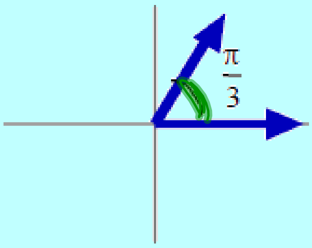
$$= \frac{95\pi}{36}$$

To find an angle *coterminal* to any given angle, add or subtract any number of full circles (± 360° or ± 2π) to the angle.

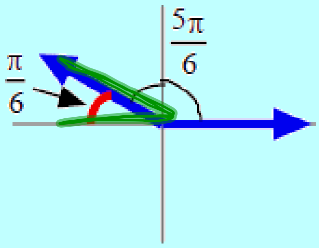
Reference Angles

A *reference angle* is an acute angle between 0 and 90° or between 0 and $\frac{\pi}{2}$ radians that represents the angle between the terminal side of an angle and the x-axis. Reference angles are always positive!

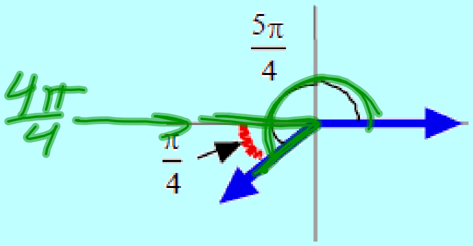
Examples:



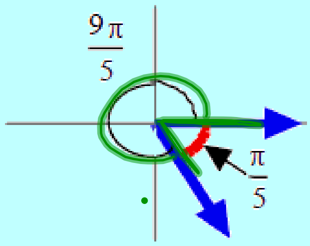
Since $\frac{\pi}{3}$ is already an acute angle, its reference angle is the same, $\frac{\pi}{3}$.



The smallest angle between the terminal side and the x-axis is $\frac{\pi}{6}$.

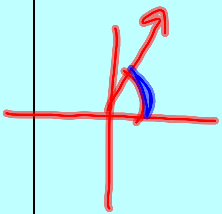
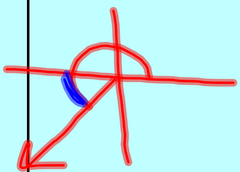
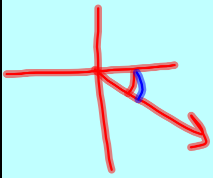
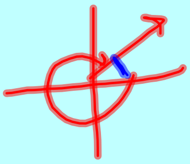


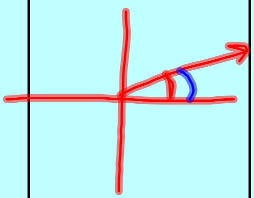
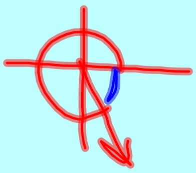
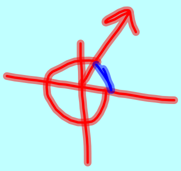
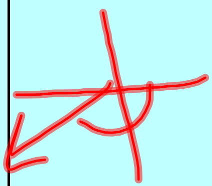
The smallest angle between the terminal side and the x-axis is $\frac{\pi}{4}$.



The smallest angle between the terminal side and the x-axis is $\frac{\pi}{5}$.

Depending on which quadrant the terminal side lies in, finding the *reference angle* is just a matter of either subtracting π, 180°, 2π or 360° from the angle or subtracting the angle from π,180°, 2π or 360°.

Angle Measure	Sketch	Reference Angle	Positive and Negative Coterminal	Convert to Radians
80°		80°	440°, -280°	$80 \cdot \frac{\pi}{180} = \frac{4\pi}{9}$
220°		40°	580°, -140°	$\frac{11\pi}{9}$
-30°		30°	330°, -390°	$-\frac{\pi}{6}$
-350°		10°	10°, -710°	$-\frac{35\pi}{18}$

Angle Measure	Sketch	Reference Angle	Positive and Negative Coterminal	Convert to Degrees
$2\pi = \frac{16\pi}{8}$ $\frac{\pi}{8}$		$\frac{\pi}{8}$	$\frac{\pi}{8} + 2\pi = \frac{17\pi}{8}$, $\frac{-15\pi}{8}$	22.5°
$2\pi = \frac{10\pi}{5}$ $\frac{8\pi}{5}$		$\frac{2\pi}{5}$	$\frac{18\pi}{5}$, $-\frac{2\pi}{5}$	288°
$-\frac{5\pi}{3}$		$\frac{\pi}{3}$	$\pi/3$, $-\frac{11\pi}{3}$	-300°
$-\frac{5\pi}{6}$		$\pi/6$	$7\pi/6$, $-\frac{17\pi}{6}$	-150°

<u>Homework Assignment:</u>

WS 4.1a Angles and their Measures

(please use key on back to check answers - no work = no credit!)