

Euler's Circuit Theorem:

- If a graph is connected and every vertex is even, then an Euler circuit exists
- If a graph has any odd vertices, then it does not exist.

Euler's Path Theorem:

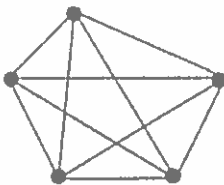
- If a graph is connected and has exactly two odd vertices, then it has at least one Euler Path. Start at one odd vertex and end at the other.
- If it has more than 2 odd vertices, then it cannot have an Euler path.

Euler's Sum of Degrees Theorem:

- The sum of the degrees of all vertices of a graph equals twice the number of edges (and is an even number)
- A graph always has an even number of odd vertices

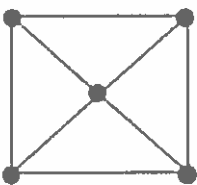
Determine whether the graph has an Euler circuit or path. Explain.

1.



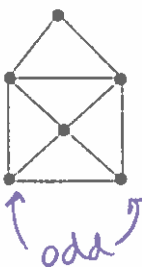
Euler Circuit

2.



none

3.



Euler Path

4.



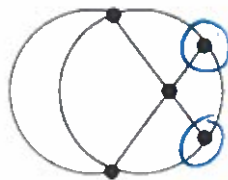
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5.



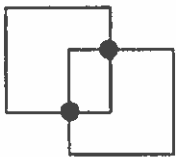
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6.



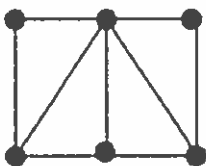
Euler Path

7.



Euler Circuit

8.



none

7. Determine whether a graph with four vertices, all of degree 1, has an Euler circuit or path.

none

8. Determine whether a graph with five vertices, all of degree 4, has an Euler circuit or path.

Circuit