SAT

A group of workers can harvest all the grapes from 10 square meters of a vineyard in $\frac{1}{2}$ minute. At this rate, how many minutes will the group need to harvest all the grapes from 100 square meters of this vineyard?

(A) 5

(B) 10

(C) 20

(D) 50

(E) 60

5.3b Logrithmic Properties

$\log_a mn =$ $\log_a m + \log_a n$

$\log_a (m/n) =$ $\log_a m - \log_a n$

$\log_a m^p =$ $p \log_a m$

Expand:

ex.1:

$\log(8xy^4) =$ $\log 8 + \log x + \log y^4$

$= \log 8 + \log x + 4\log y$

ex.2:

$\ln\left(\sqrt{x^2+5}/x\right) =$ $\ln\sqrt{x^2+5} - \ln x$

$= \ln(x^2+5)^{\frac{1}{2}} - \ln x$

$= \frac{1}{2}\ln(x^2+5) - \ln x$

In Exercises 1–12, assuming x and y are positive, use properties of logarithms to write the expression as a sum or difference of logarithms or multiples of logarithms.

1. $\ln 8x$

2. $\ln 9y = \ln 9 + \ln y$

3. $\log \frac{3}{x}$

4. $\log \frac{2}{y}$ $\log 2 - \log y$

5. $\log_2 y^5$

6. $\log_2 x^{-2}$ $-2\log_2 x$

7. $\log x^3 y^2$

8. $\log xy^3$ $\log x + 3\log y$

9. $\ln \frac{x^2}{y^3}$

10. $\log 1000x^4$ $\log 1000 + 4\log x$

11. $\log \frac{\sqrt{x}}{\sqrt[3]{y}}$

12. $\ln \frac{\sqrt[3]{x}}{\sqrt[3]{y}}$ $\frac{1}{3}\ln x - \frac{1}{3}\ln y$

Condense:

ex.1:

$5\ln x - 2\ln(xy) =$

$= \ln x^5 - \ln (xy)^2$

$= \ln \frac{x^5}{(xy)^2} = \ln \frac{x^5}{x^2y^2}$

$= \ln \frac{x^3}{y^2}$

In Exercises 13–22, assuming x, y, and z are positive, use properties of logarithms to write the expression as a single logarithm.

13. $\log x + \log y$
14. $\log x + \log 5$ $\log 5x$
15. $\ln y - \ln 3$
16. $\ln x - \ln y$ $\ln \frac{x}{y}$
17. $\frac{1}{3} \log x$
18. $\frac{1}{5} \log z$ $\log z^{\frac{1}{5}} = \log \sqrt[5]{z}$
19. $2 \ln x + 3 \ln y$
20. $4 \log y - \log z$ $\log \frac{y^4}{z}$
21. $4 \log (xy) - 3 \log (yz)$
22. $3 \ln (x^3y) + 2 \ln (yz^2)$ $\ln (x^3y)^3 (yz^2)^2$ $= \ln x^9 y^5 z^4$

Change of Base Formula

$$\log_4 7 = y$$

$$y = \frac{\log 7}{\log 4}$$

Formula: $a \neq 1, b \neq 1$

$$\log_b x = \frac{\log_a x}{\log_a b}$$

More Generally:

$$\log_b x = \frac{\log x}{\log b}$$

or

$$\log_b x = \frac{\ln x}{\ln b}$$

examples:

a) $\log_3 16 =$ $\frac{\log 16}{\log 3} \approx 2.52$

b) $\log_6 10 =$ $\frac{\ln 10}{\ln 6} \approx 1.29$

c) $\log_{1/2} 2 =$ $\frac{\log 2}{\log \frac{1}{2}} = -1$

exs. page 363 (24-34even)

In Exercises 23–28, use the change-of-base formula and your calculator to evaluate the logarithm.

23. $\log_2 7$ $\frac{\log 19}{\log 5} \approx 1.83$
24. $\log_5 19$
25. $\log_8 175$
26. $\log_{12} 259$ $\frac{\log(259)}{\log(12)} \approx 2.24$
27. $\log_{0.5} 12$
28. $\log_{0.2} 29$ $\frac{\log 29}{\log 0.2} \approx -2.09$

In Exercises 29–32, write the expression using only natural logarithms.

29. $\log_3 x$
30. $\log_7 x$ $\frac{\ln x}{\ln 7}$
31. $\log_2 (a + b)$
32. $\log_5 (c - d)$ $\frac{\ln(c-d)}{\ln 5}$

In Exercises 33–36, write the expression using only common logarithms.

33. $\log_2 x$ $\frac{\log x}{\log 4}$
34. $\log_4 x$
35. $\log_{1/2} (x + y)$
36. $\log_{1/3} (x - y)$ $\frac{\log(x-y)}{\log 1/3}$

Homework Assignment:
pages 363-364
(23-37 odd, 79-90 all)