IB Math Studies / Algebra 2

4.5 Rational Functions and Asymptotes

In the past, you may have studied negative exponents. They are used to represent variables that are in the denominator of a fraction.

Fraction form	Power form	Fraction form	Power form
$\frac{1}{x^2}$			$2x^{-1}$
$\frac{3}{x^2}$		$\frac{2}{3x^3}$	
$\frac{1}{2x^2}$			$4x^{-3}$
$\frac{1}{x}$		$\frac{4}{x}$	

Polynomial functions can also contain terms with negative exponents.

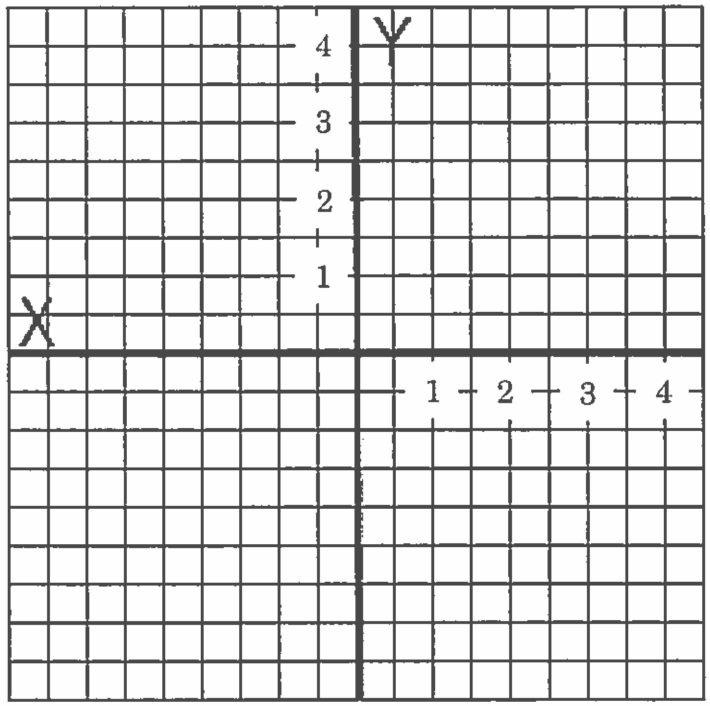
$f(x)$ in Fraction form	$f(x)$ in Power form
$f(x) = 3x^2 + \frac{2}{x}$	
	$f(x) = 2x + 50x^{-2}$
	$f(x) = 2x^2 + 2000x^{-1}$
$f(x) = 21 + \frac{79}{x}$	

If a function has a variable in the denominator, then it is called a rational function.

These functions can have vertical and horizontal asymptotes.

1. $f(x) = \frac{1}{x}$

x	$f(x) = \frac{1}{x}$
3	
2	
1	
$\frac{1}{2}$	
0	
$-\frac{1}{2}$	
-1	
-2	
-3	



Vertical Asymptote:

Domain:

Horizontal Asymptote:

Range:

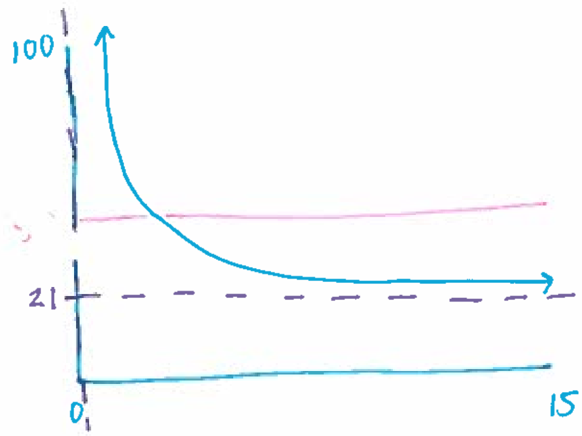
What happens to y as x gets closer to zero?

What happens to y as x gets larger and larger?

2. The temperature of water as it cools to room temperature is modeled by the following function, where x is the time in minutes and $f(x)$ represents the temperature in $°C$.

$$f(x) = 21 + \frac{79}{x}, \qquad x \neq 0$$

a) Use your GDC to sketch the graph of the function for $0 < x \leq 15$ and $0 < y \leq 100$. Indicate the asymptotes with dashed lines.



b) Calculate the temperature of the water after 10 minutes.

$f(10) = 28.9°C$

c) How many minutes does it take for the temperature to cool down to 50 $°C$? Show this on your graph in part a).

$x = 2.72$ minutes.

d) Write down the equation of the vertical asymptote.

$x = 0$

e) Write down the equation of the horizontal asymptote.

$y = 21$

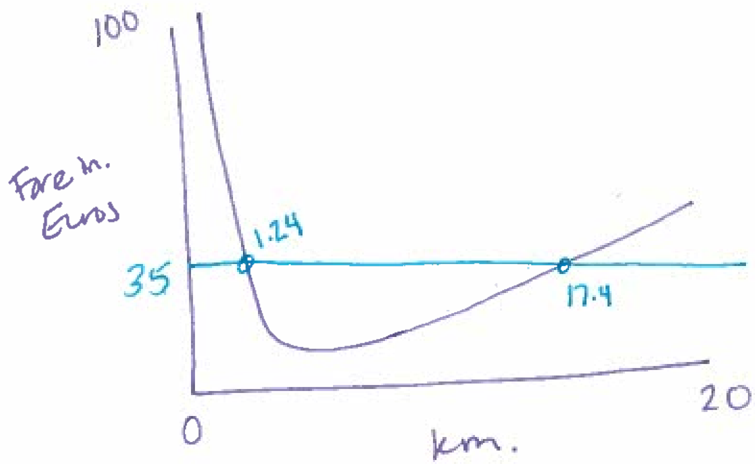
f) Write down the room temperature.

21°C

3. A taxi company's fares depend on the distance, in kilometers, traveled. The fares are calculated using the formula below, where x is number of kilometers traveled and $f(x)$ is the fare in euros.

$$f(x) = 2x + \frac{50}{x^2}, \qquad x \neq 0$$

a) Sketch the graph of the function for $0 < x \leq 20$.



b) Find the cost of a journey of 10 kilometers.

F(10) = 20.5 Euros.

c) Two different journeys will both cost 35 euros. Find the distances of these two journeys. Show this on your graph in part a).

17.4 km or 1.24 km.

d) Find the distance traveled that gives the cheapest fare.

X= 3.68 km

y=(11.05 Euros)

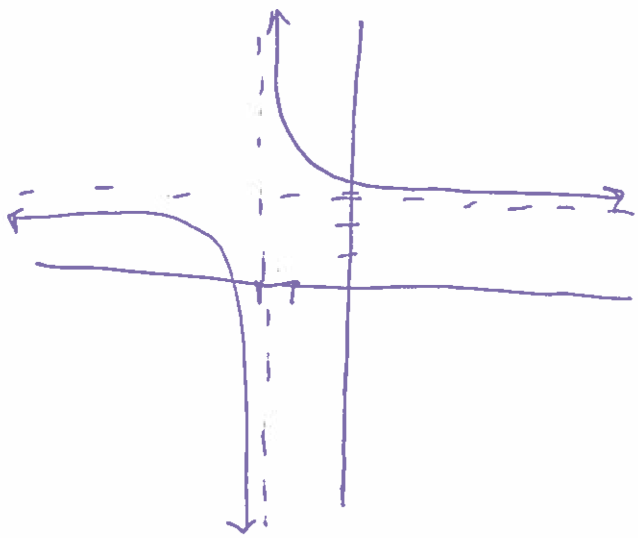
e) Write down the domain of the function.

$x \in \mathbb{R},\ x \neq 0$

f) Write down the range of the function.

$y \geq 11.05$

4. a) Use your GDC to sketch the graph of $f(x) = 3 + \frac{6}{x+2}$, $x \neq 0$,
for $-10 \leq x \leq 10$.



b) Find the value of $f(8)$.

$f(8) = 3.6$

c) Find the x-intercept.

$(-4, 0)$

d) Write down the equations of the vertical and horizontal asymptotes.

$x = -2$ $y = 3$

e) Write down the domain of the function.

$x \in \mathbb{R},\ x \neq -2$

f) Write down the range of the function.

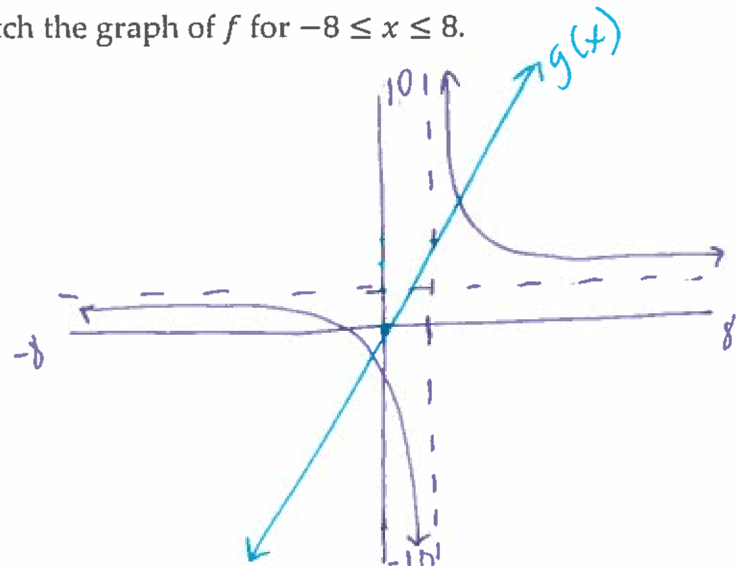
$y \in \mathbb{R},\ y \neq 3$

5. The functions f and g are defined by

$$f(x) = 1 + \frac{4}{x-1}, \qquad x \neq 0$$

$$g(x) = 3x$$

a) Sketch the graph of f for $-8 \leq x \leq 8$.



b) Write down the equations of the vertical and horizontal asymptotes of the function f.

VA: $x = 1$

HA: $y = 1$

c) Sketch the graph of g on the same axes.

d) Find the solutions to $f(x) = g(x)$. $x = -0.535$ or 1.87

e) Write down the range of $f(x)$.

$y \in \mathbb{R}, y \neq 1$

f) Write down the domain of $g(x)$.

$x \in \mathbb{R}, x \neq 1$