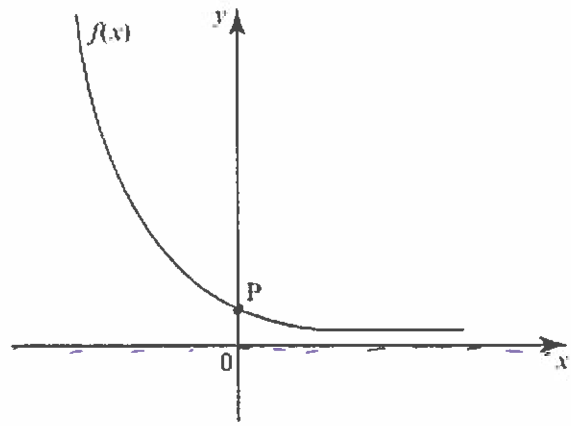
$F(x) = a^x$ Key

B Math Studies / Algebra 2

4.4c Exponential Functions: Real-Life Applications and IB Problems

1. The following diagram shows part of the graph of an exponential function $f(x) = 3a^{-x}$, where $x \in \mathbb{R}$.



a) What is the range of f?

$y > 0$

b) Write down the coordinates of the point P.

$(0,3)$

$f(0) = 3a^{-0} = 3(1) = 3$

c) What happens to the values of $f(x)$ as the members in its domain increase in value?

as x-values increase

Values of $f(x)$ decrease.

2. The equation $M = 90 \times 2^{-t/20}$ gives the amount, in grams, of radioactive material held in a laboratory over t years.

a) What was the original mass of the radioactive material?

$t=0$

$M = 90 \times 2^{-0/20}$

$= 90 \times 2^0 = 90 \times 1 = 90g$

The table lists some values for M.

t	60	80	100
M	11.25	v	2.8125

b) Find the value of v.

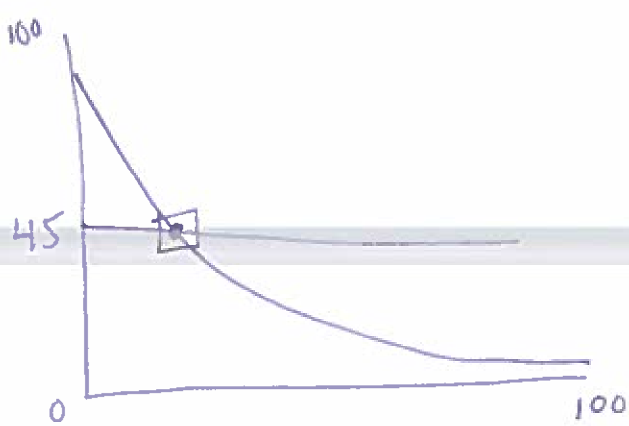
$M = 90 \times 2^{-80/20}$

$v = 90 \times 2^{-4}$

$v = 5.625g$

c) Find the number of years it would take for the radioactive material to have a mass of 45 grams. Sketch the graph you used to find this answer.

$\underset{Y_1}{45} = \underset{Y_2}{90 \times 2^{-\frac{t}{20}}}$



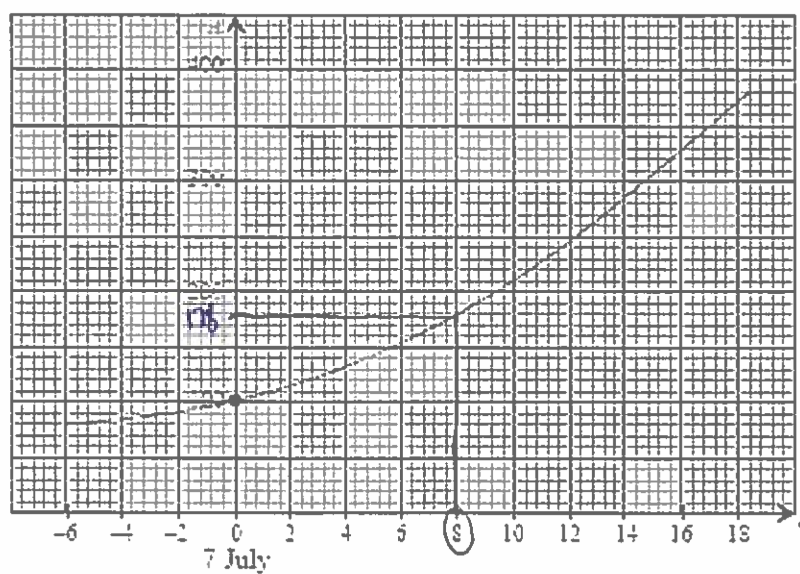
$t = 20$ years.

3. The area, A m², of a fast-growing plant is measured at noon (12:00) each day. On 7 July the area was 100 m². Every day, the plant grew by 7.5%.

a) Write an exponential equation to represent the area of the plant t days after 7 July. Use the formula $A = P_0(1 + r)^t$. $r = 0.075$

$$A = 100(1+0.075)^t$$

The graph of the area is shown below.



b) What does the graph represent when t is negative?

height of plant before July 7

c) Use the graph to find the value of t when $A = 178$.

$t = 8$ days.

d) Use your equation from part a) to calculate the area covered by the plant at noon on 28 July.

28 July $\rightarrow$ $t = 21$

$$A = 100(1+0.075)^{21}$$

$$= 456.64 \text{ m}^2$$

4. The number (n) of bacteria in a colony after h hours is given by the formula $n = 1200(3^{0.25h})$. Initially, there are 1200 bacteria in the colony.

a) Complete the table below. Give your answers to the nearest hundred.

time in hours (h)	0	1	2	3	4
no. of bacteria (n)	1200	1600	2100	2700	3600

$1200(3^{0.25(1)})$ $1200(3^{0.25(4)})$

b) On graph paper, draw the graph of the above function. Use a scale of 3 cm to represent 1 hour on the horizontal axis and 4 cm to represent 1000 bacteria on the vertical axis. Label the graph clearly.

See graph on next pg.

c) Use your graph to answer each of the following. Show your method clearly on the graph.

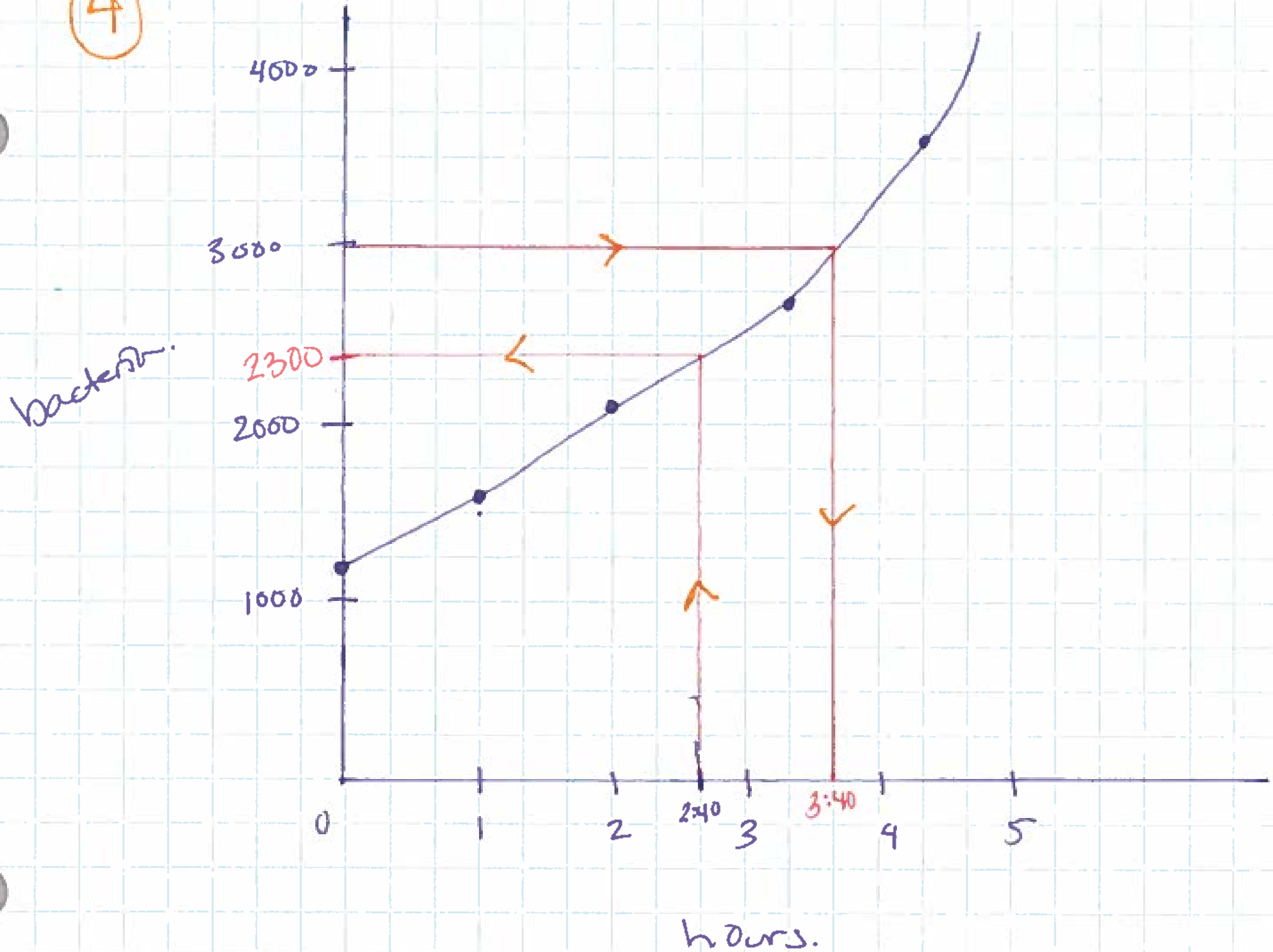
i) How many bacteria would there be after 2 hours and 40 minutes? Give your answer to the nearest hundred bacteria.

2300

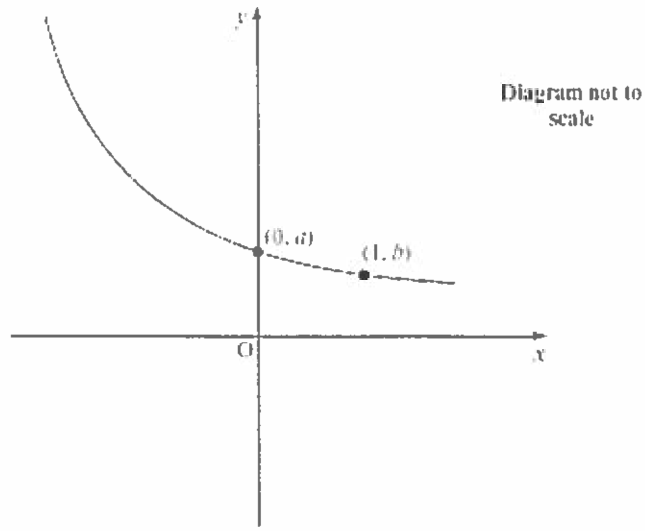
ii) After how long will there be approximately 3000 bacteria? Give your answer to the nearest 10 minutes.

3hrs. 40mins.

4



5. The following diagram shows the graph of $y = 3^{-x} + 2$. The curve passes through the points $(0, a)$ and $(1, b)$.



a) Find the value of

i) a

$y = a$

$a = 3^{-0} + 2 = 1 + 2 = 3$

ii) b

$y = b$

$b = 3^{-1} + 2 = \frac{1}{3} + 2 = 2.33$

b) Write down the equation of the asymptote to this curve.

$y = 2$

c) Write down the domain and range of this curve.

D: $x \in \mathbb{R}$

R: $y > 2$

6. The diagram below shows a part of the graph $y = a^x$. The graph crosses the y-axis at the point P. The point $Q(4,16)$ is on the graph.

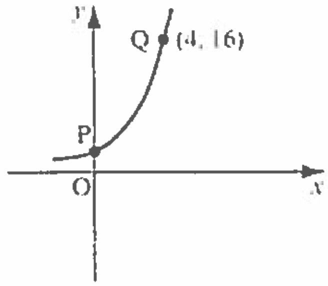


diagram not to scale

Find

a) the coordinates of the point P.

$y = a^x$

$y = a^0$

$y = 1$

P : (0,1)

Q : (4,16)

b) the value of a.

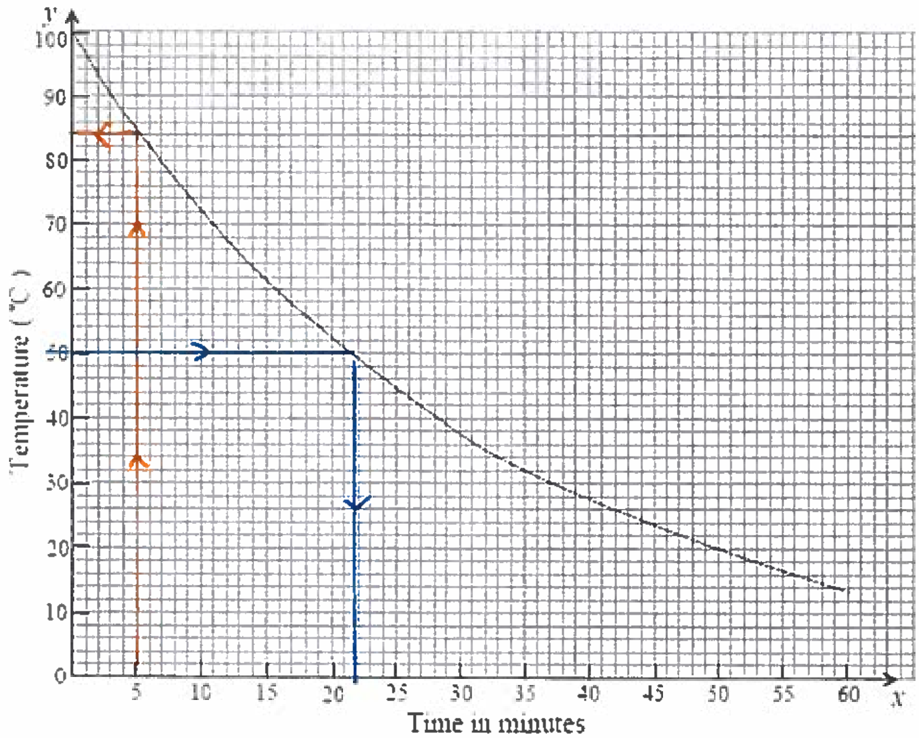
$y = a^x$

$16 = a^4$

$\sqrt[4]{16} = a$

$a = 2$

7. The graph below shows the temperature of a liquid as it is cooling.



a) Write down the temperature after 5 minutes.

84°C

b) After how many minutes is the temperature 50°C?

23 minutes

The equation of the graph for all positive x can be written in the form

$$y = 100(5^{-0.02x}).$$

c) Calculate the temperature after 80 minutes.

$y = 100(5^{-0.02 \cdot 80}) = 7.61°C$

d) Write down the equation of the asymptote to the curve.

$y = 0$

8. A function is represented by the equation $f(x) = 3(2)^x + 1$.

The table of values of $f(x)$ for $-3 \leq x \leq 2$ is given below.

x	-3	-2	-1	0	1	2
$f(x)$	1.375	1.75	a	4	7	b

a) Calculate the values for a and b.

$F(-1) = 3(2)^{-1} + 1$

$a = 3(\frac{1}{2}) + 1 = 2.5$

$f(2) = 3(2)^2 + 1$

$b = 12 + 1 = 13$

b) On graph paper, draw the graph of $f(x)$, for $-3 \leq x \leq 2$, taking 1 cm to represent 1 unit on each axis.

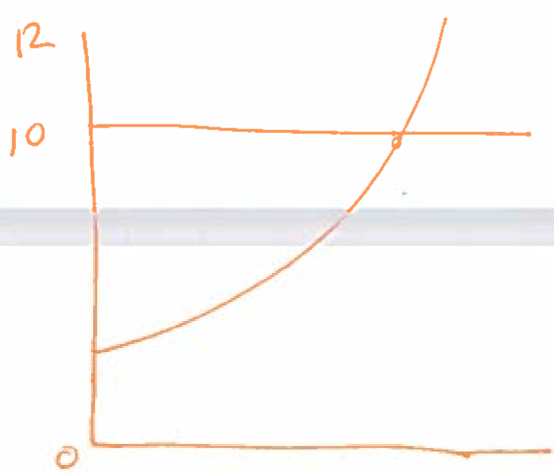
The domain of the function $f(x)$ is the real numbers, $\mathbb{R}$.

c) Write down the range of $f(x)$.

$y > 1$

d) Using your graph, or otherwise, find the approximate value for x when $f(x) = 10$.

$10 = 3(2)^x + 1$



$x = 1.6$