

# Ch.4b Exponential and Rational Models

In 1984, the world population was about 4.8 billion. It was growing at a rate of 2% each year. What was the population in 1985? In 1990? What will it be in 2050?

$$(4.8)(0.02) + 4.8 = \boxed{\phantom{000}}$$

$$\boxed{\phantom{000}}(0.02) + \boxed{\phantom{000}}$$

When a population is changing at a constant percentage rate,  $r$ , each year, then the model for growth or decay is

$$P(t) = P_0(1+r)^t$$

where

$P_0$  is the initial value at time  $t = 0$

$r$  is the growth / decay rate expressed as a decimal

$t$  is the time in years.

$$P(t) = 4.8(1+0.02)^t$$

$$1985, t=1 \rightarrow P(1) = 4.8(1+0.02)^1 \\ = 4.896$$

$$1990, t=6 \rightarrow P(6) = 4.8(1+0.02)^6 \\ = 5.406$$

$$2050, t=66 \quad P(66) = 17.756$$

billion  
people

1. **Interpreting Population Functions:** Tell whether the population model is an exponential growth function or exponential decay function. Then identify the initial population and the constant percentage rate of growth or decay.

$$P(t) = P_0(1+r)^t$$

a)  $P(t) = 15 \cdot 1.0135^t = 15 \cdot (1 + 0.0135)^t$  Increasing!  
 $P_0 = 15$   $r = 0.0135$   
 $1.35\%$

b)  $P(t) = 35 \cdot 0.93^t$

b)  $P(t) = 35(1 - 0.07)^t$  decreasing  $r = 0.07$   
 $P_0 = 35$   $-7\%$

c)  $g(x) = 43 \cdot (1 + 0.05)^x + 7$  increasing  $r = 0.05$   
 $P_0 = 50$   $5\%$   
 $(43 + 7)$

2. **Writing Population Functions:** Write an exponential function that models the given conditions.

$$P(t) = P_0(1+r)^t$$

- a) A population begins at 12, and increases at a rate of 12% per year

$$P(t) = 12(1+0.12)^t$$

- b) A population begins at 5, and decreases at a rate of 8% per year

$$P(t) = 5(1-0.08)^t$$

- c) A population begins at 502,000, and decreases at a rate of 1.7% per year

$$P(t) = 502000(1-0.017)^t$$

In 1984, the world population was about 4.8 billion. It was growing at a rate of 2% each year.

What was the population in 1985?

In 1990?

What will it be in 2050?

Homework Assignment:  
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