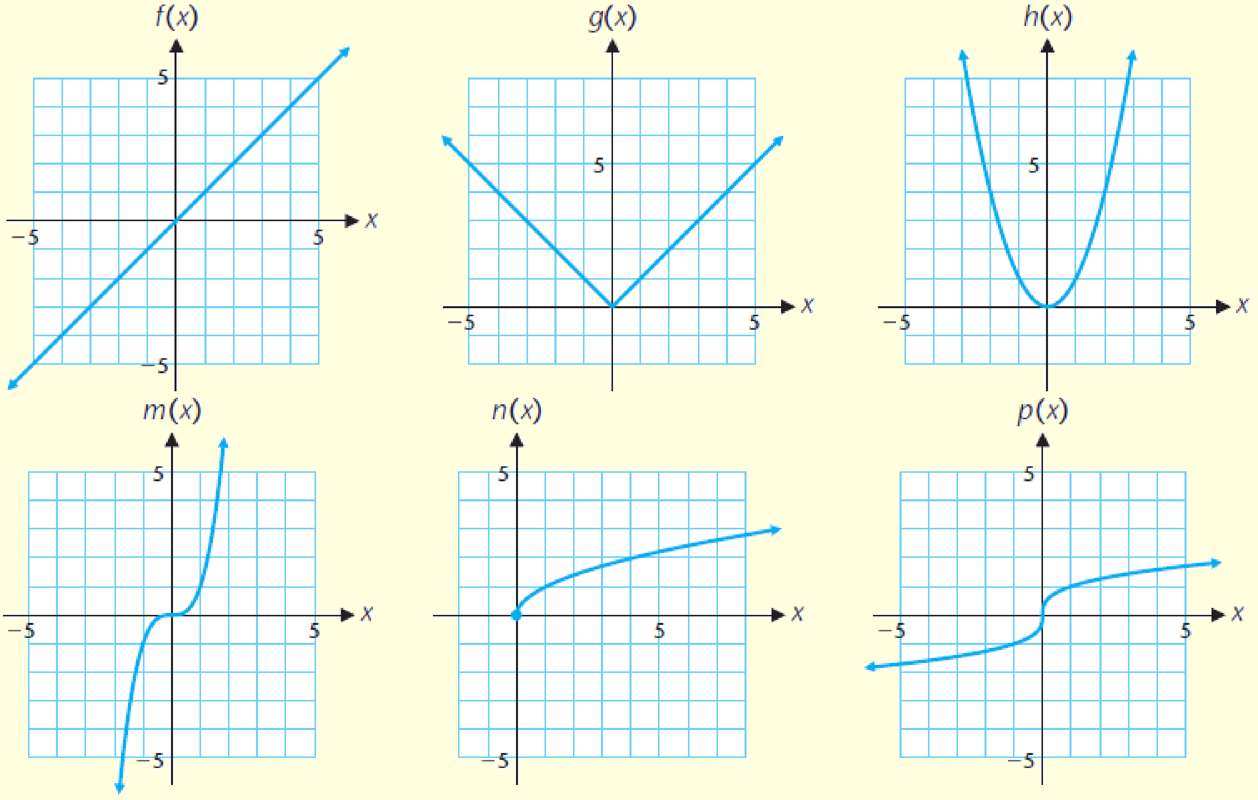
SAT:

If $y = \frac{(x+1)(x-2)}{(x+3)(x-4)}$, for which values of x is y NOT defined?

3-3b Non-Rigid Transformations (dilations)

Review of Basic Functions...



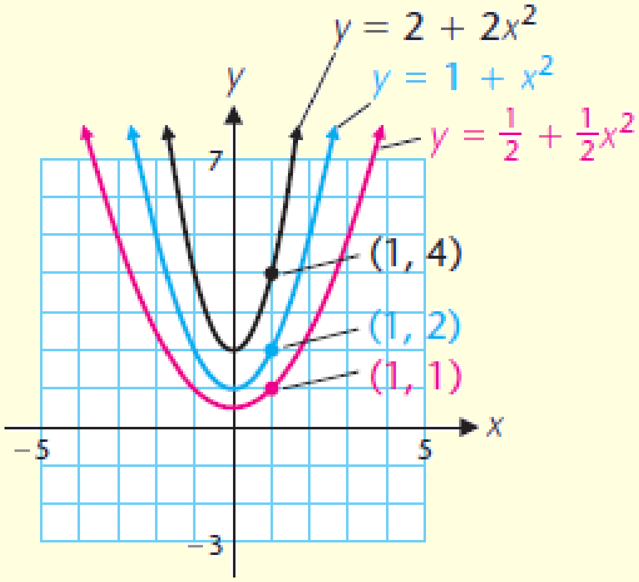
Putting it all together

Consider the basic graph of the function: y = f(x)
All of the translations can be expressed in the form:
y = a * f [b (x-c)] + d

	Vertical	Horizontal
Scale	a	b
Shift	d	c
	acts normally	acts inversely

Let $f(x) = 1 + x^2$. $y_2 = 2(1+x^2)$ $y_3 = \frac{1}{2}(1+x^2)$

(A) How are the graphs of $y = 2f(x)$ and $y = \frac{1}{2}f(x)$ related to the graph of $y = f(x)$? Confirm your answer by graphing all three functions in the same coordinate system.

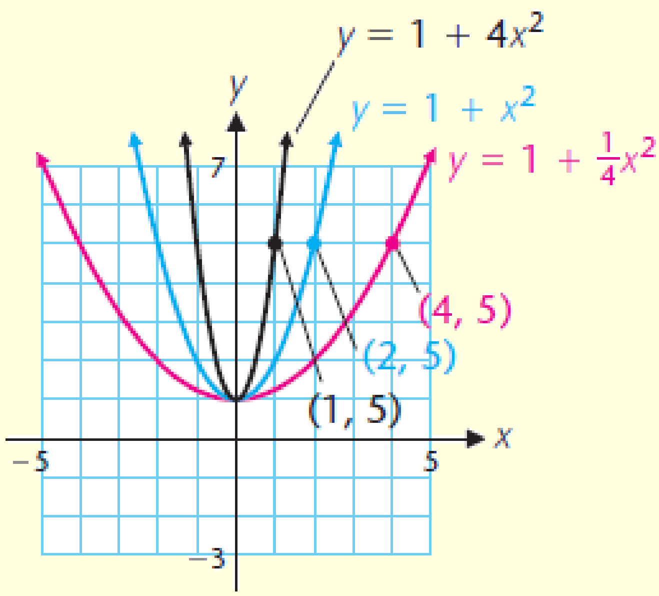


(a) Vertical stretching and shrinking

Let $f(x) = 1 + x^2$.

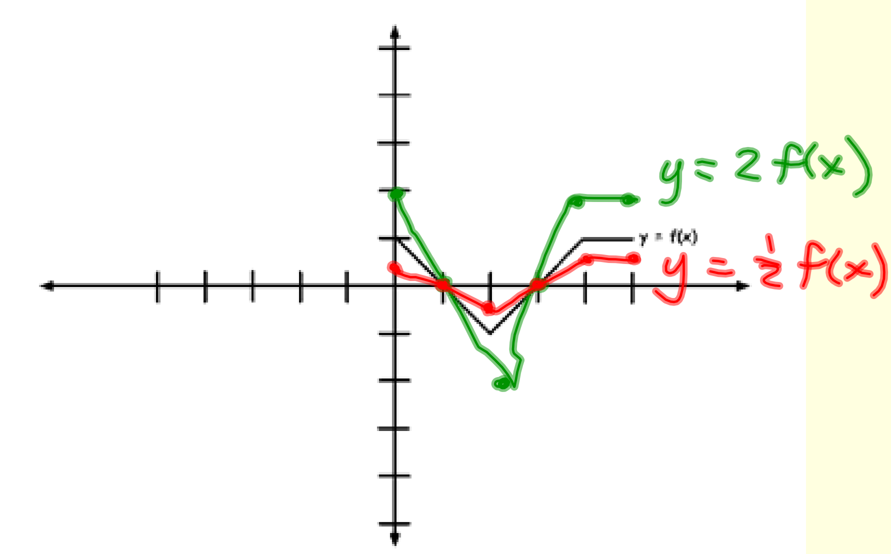
$y_2 = 1+(2x)^2$ $y_3 = 1+(\frac{1}{2}x)^2$

(B) How are the graphs of $y = f(2x)$ and $y = f(\frac{1}{2}x)$ related to the graph of $y = f(x)$? Confirm your answer by graphing all three functions in the same coordinate system.

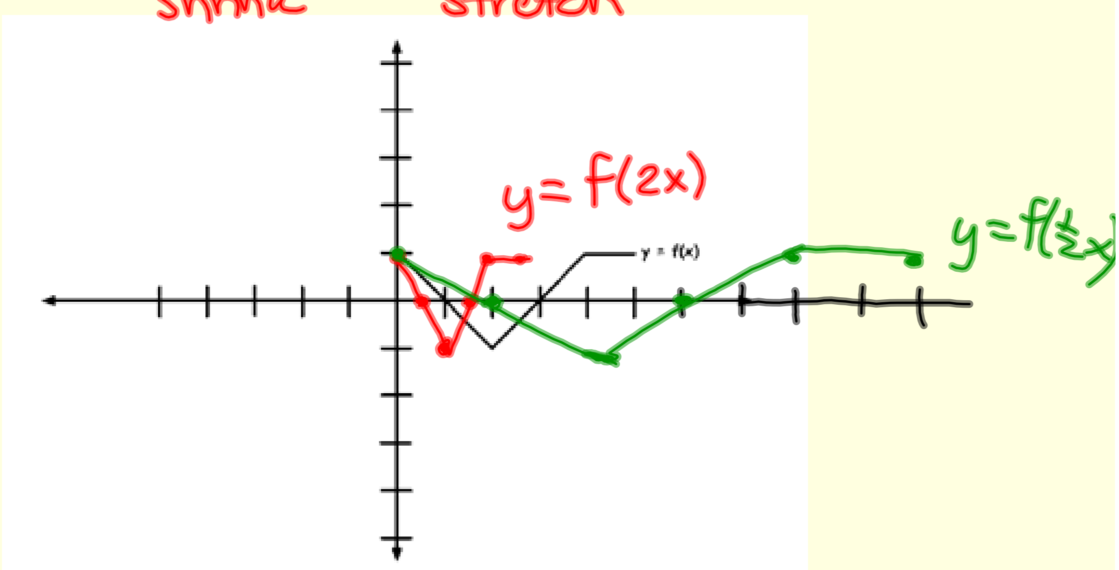


(b) Horizontal stretching and shrinking

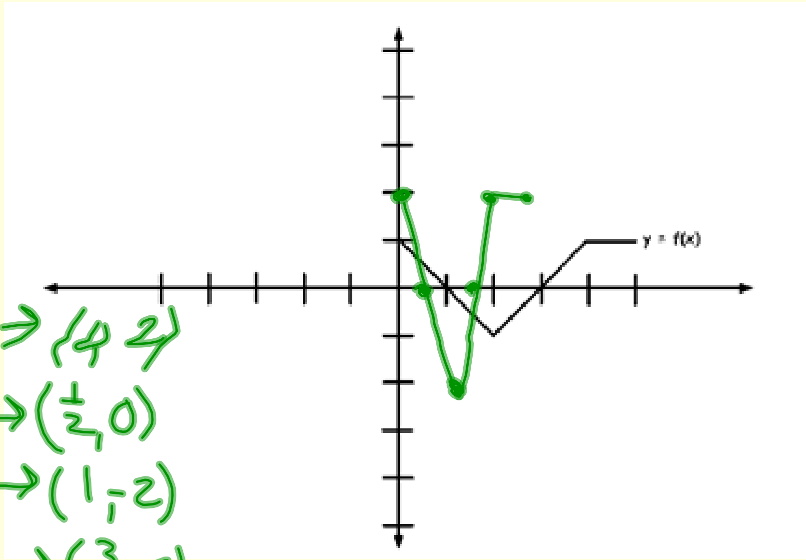
Graph 2*f*(*x*) and (1/2)*f*(*x*) on the axes



Graph $f(2x)$ and $f(1/2x)$ on the axes



Graph $2f(2x)$ on the axes



$(0, 1) \to (0, 2)$

$(1, 0) \to (\frac{1}{2}, 0)$

$(2, -1) \to (1, -2)$

$(3, 0) \to (\frac{3}{2}, 0)$

$(4, 1) \to (2, 2)$

$(5, 1) \to (5/2, 2)$

If $A > 1$, is the graph of $y = f(Ax)$ a horizontal stretch or a horizontal shrink of the graph of $y = f(x)$? What if $0 < A < 1$?

Basic Ideas of Transformations of Functions

Exploration: Vertical Dilations

Exploration: Horizontal Dilations

3-3 Graphical Transformations (non-Rigid)

Interactive Quiz: Identifying Transformations

Interactive Quiz: Writing Transformations

Definition

For the base function $f(x)$ and a constant $k > 0$, the function given by

$$g(x) = k\,f(x),$$

can be sketched by vertically stretching $f(x)$ by a factor of k if $k > 1$

or

by vertically shrinking $f(x)$ by a factor of k if $0 < k < 1$.

<-- Vertical Stretches/Shrinks

Horizontal --> Stretches/Shrinks

Definition

For the base function $f(x)$ and a constant k, where $k > 0$ and $k \neq 1$, the function given by

$$g(x) = f(kx),$$

can be sketched by horizontally shrinking $f(x)$ by a factor of $1/k$ if $k > 1$

or

by horizontally stretching $f(x)$ by a factor of $1/k$ if $0 < k < 1$.

It is useful to be able to interpret a function as a dilation of a simpler “parent function”.

Understanding the behavior of

$$g(x) = 3(x-1)^2 + 5$$

is easier when you perceive this as the graph of

$$f(x) = x^2$$

dilated vertically by a factor of 3, then translated horizontally by +1 and vertically by +5.

Finding Equations for Stretches and Shrinks:

Let $y_1 = f(x) = x^3 - 16x$

Find equations for the non-rigid transformations of $f(x)$.

(a) y_2 is a vertical stretch of y_1 by a factor of 3.

$y_2 = 3(x^3 - 16x)$
$= 3x^3 - 48x$

(b) y_3 is a horizontal shrink of y_1 by a factor of 1/2.

$y_3 = (2x)^3 - 2(16x)$
$= 8x^3 - 32x$

*Support graphically...

Combining Transformations in Order

(a) The graph of $y = x^2$ undergoes the following transformations, in order. Find the equation of the graph that results.

- a horizontal shift 2 units to the right
- a vertical stretch by a factor of 3
- a vertical translation 5 units up

$y = 3(x-2)^2 + 5$

(b) Apply the transformations in (a) in the opposite order and find the equation of the graph that results.

$y = 3((x-2)^2 + 5)$

GRAPH TRANSFORMATIONS (SUMMARY)

Vertical Shift [Fig. 11(a)]:

$$y = f(x) + k \quad \begin{cases} k > 0 & \text{Shift graph of } y = f(x) \text{ up } k \text{ units} \\ k < 0 & \text{Shift graph of } y = f(x) \text{ down } |k| \text{ units} \end{cases}$$

Horizontal Shift [Fig. 11(b)]:

$$y = f(x + h) \quad \begin{cases} h > 0 & \text{Shift graph of } y = f(x) \text{ left } h \text{ units} \\ h < 0 & \text{Shift graph of } y = f(x) \text{ right } |h| \text{ units} \end{cases}$$

Vertical Stretch and Shrink [Fig. 11(c)]:

$$y = Af(x) \quad \begin{cases} A > 1 & \text{Vertically stretch the graph of } y = f(x) \text{ by multiplying each } y \text{ value by } A \\ 0 < A < 1 & \text{Vertically shrink the graph of } y = f(x) \text{ by multiplying each } y \text{ value by } A \end{cases}$$

Horizontal Stretch and Shrink [Fig. 11(d)]:

$$y = f(Ax) \quad \begin{cases} A > 1 & \text{Horizontally shrink the graph of } y = f(x) \text{ by multiplying each } x \text{ value by } \frac{1}{A} \\ 0 < A < 1 & \text{Horizontally stretch the graph of } y = f(x) \text{ by multiplying each } x \text{ value by } \frac{1}{A} \end{cases}$$

Reflection [Fig. 11(e)]:

$y = -f(x)$ Reflect the graph of $y = f(x)$ through the x axis

$y = f(-x)$ Reflect the graph of $y = f(x)$ through the y axis

$y = -f(-x)$ Reflect the graph of $y = f(x)$ through the origin

Homework Assignment:

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(7,9,21,23,39-43 odd,45,53,61,67,77)