Discrete Math: 2.2.1 The Banzhaf Power Index
Date:

Name: Key
Period:

Coalition:

a set of players who might join forces + vote the same way.

Number of Coalitions:

$2^n - 1$ (n is total number of players)

Grand Coalition:

Consists of all players.

Winning Coalition:

Coalitions with enough votes to win.

Critical Player:

Coalitions must have this player in order to win.

$W - w < q$

W = weight of coalition, w = weight of critical player

The Banzhaf power index:

critical count for $P_1 = B_1$

$T = B_1 + B_2 + B_3 + \ldots B_N$

B. Power Index = $\frac{B_1}{T}$ (β_1)

The Banzhaf power distribution:

Complete list of power indexes $\beta_1, \beta_2, \beta_3 \ldots \beta_N$

(sum of all β's is 1 or 100%)

Consider the weighted voting system [18: 12, 10, 7, 1].

1. What is the weight of the coalition of P_1 and P_3?

$P_1 + P_3 = 19$

2. Write down all winning coalitions.

$P_1 + P_2 = 22$

$P_1 + P_3 = 19$

$P_1 + P_2 + P_3 = 29$

$P_2 + P_3 + P_4 = 18$

$P_1 + P_2 + P_4 = 23$

$P_1 + P_3 + P_4 = 20$

$P_1 + P_2 + P_3 + P_4 = 30$

3. Which players are critical in the coalition $\{P_1, P_2, P_3\}$? $W - w \geq q$

$W = 29$

P_1 bc $29 - 12 = 17$

4. Find the Banzhaf power distribution of this weighted voting system.

$\{\underline{P_1} + \underline{P_2}\}$

$\{\underline{P_1} + \underline{P_3}\}$

$\{\underline{P_1} + P_2 + P_3\}$

$\{\underline{P_1} + \underline{P_2} + P_4\}$

$\{\underline{P_2} + \underline{P_3} + \underline{P_4}\}$

$\{\underline{P_1} + \underline{P_3} + P_4\}$

$\{P_1 + P_2 + P_3 + P_4\}$

Player	Critical #	B.P.I
P_1	5	$\frac{5}{12} = 41.7\%$
P_2	3	$\frac{3}{12} = 25\%$
P_3	3	$\frac{3}{12} = 25\%$
P_4	1	$\frac{1}{12} = 8.3\%$
Total:	12	