Discrete Math: 2.1.1 An Introduction to Weighted Voting
Date:

Name:
Period: Key

Weighted Voting System:
formal voting arrangement where voters are not equal in terms of # of votes they control

Motions:
yes-no votes

The Players: voters (N - # of voters, $P_1, P_2, P_3 \dots P_N$ - player 1, etc.)

The Weights: number of votes a player controls ($w_1, w_2, w_3 \dots$)

$V = w_1 + w_2 + w_3 + \dots + w_N$ ← total votes

The Quota:
minimum number of votes required to pass a motion

Dictator: a player who's weight is bigger than or equal to the quota (must be P_1 - if P_1 is not a dictator then no dictator exists)

Dummies: players who have no power (all players except dictator)

Veto Power: a player has veto power if a motion cannot pass unless the player votes in favor of the motion. ($w < q$ and $V - w < q$)

Consider the weighted voting system [87: 33, 31, 23, 20, 15, 7].

1. Find the total number of players.
   6
2. Find the total number of votes.
   129
3. Find the weight of P_2.
   31
4. Find the minimum percentage of the votes needed to pass a motion (rounded to the next whole percent).
   68%

Consider the weighted voting system [q: 16, 13, 8, 6, 2]. $\frac{V}{2} < q \leq V$

5. What is the smallest value that the quota q can take?

$\frac{45}{2} = 22.5$ 23

6. What is the largest value that the quota q can take?

45

7. What is the value of the quota if at least two-thirds of the votes are required to pass a motion?

$45(\frac{2}{3}) =$ 30

8. What is the value of the quota if more than two-thirds of the votes are required to pass a motion?

31 or more.

A committee has five members (P_1, P_2, P_3, P_4, and P_5). In this committee P_1 has three times as many votes as P_2; P_2 has twice as many votes as P_3, P_3 has twice as many votes as P_4; P_4 and P_5 have the same number of votes. The quota is 109. For each of the given definitions of the quota, describe the committee using the notation [q: w_1, w_2, w_3, w_4, w_5]. (Hint: Write the weighted voting system as [109: 3x, 2x, 2x, x, x].

9. The quota is defined as a simple majority of the votes.

$109 - 1 = 108$

$\times 2$

$V = 216$

$3x + 2x + 2x + x + x = 216$

$x = 24$

[109: 72, 48, 48, 24, 24]

10. The quota is defined as more than two-thirds of the votes.

$\frac{2}{3}V = 108$

$V = 162$

$3x + 2x + 2x + x + x = 162$

$x = 18$

[109: 54, 36, 36, 18, 18]

11. The quota is defined as more than three-fourths of the votes.

$\frac{3}{4}V = 108$

$V = 144$

[109: 48, 32, 32, 16, 16]

$3x + 2x + 2x + x + x = 144$

$9x = 144$

$x = 16$