Section 13.2: General Angles and Radian Measure

This section is divided up into 5 mini lessons that will all fit together in the end like a big puzzle.

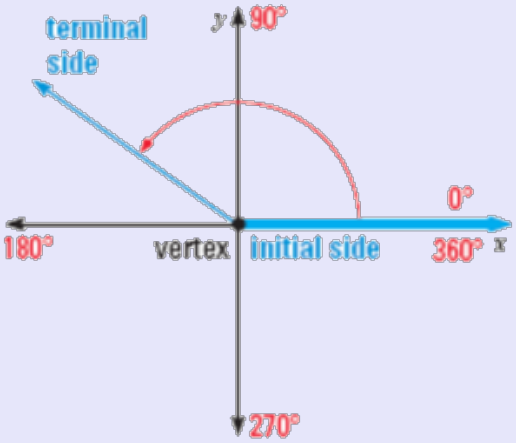
You might feel out of your comfort zone for a while, but stick with it...and study!

A. Angles in Standard Position

- In the last section, we just worked with acute angles in triangles.

- Now, we will branch out to angles with a measure of any real number.

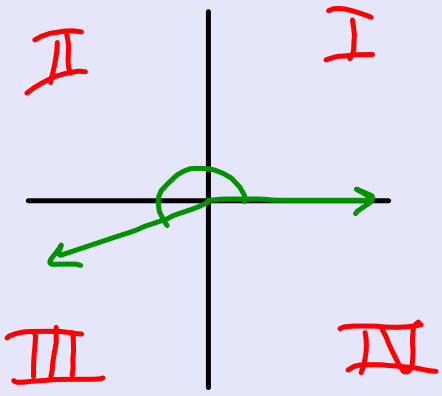
- This requires some new vocabulary terms.



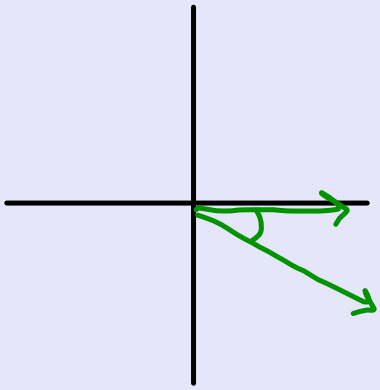
We will start "mapping" angles on a coordinate plane.

Ex. 1: Let's draw some angles in standard position.

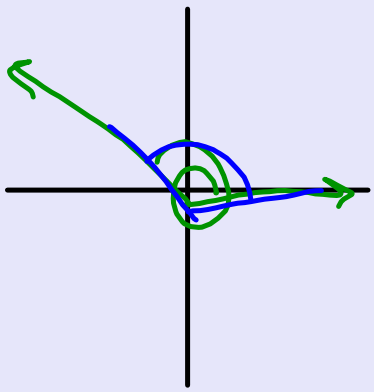
a. 210°



b. -45°



c. 510°



$$\begin{array}{r} 510 \\ -360 \\ \hline 150^\circ \end{array}$$

B. Coterminal Angles

Two angles are **coterminal** if their terminal sides coincide.

In the previous example, 150° and 510° are coterminal.

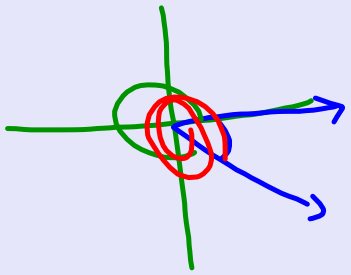
What is the numeric relationship between these two measures?

Ex. 2: Find one positive and one negative angle that is coterminal with the given angle.

a. -60°

$-60 + 360 = 300°$

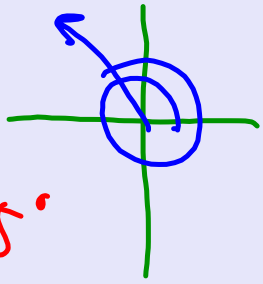
$-60 - 360 = -420°$



b. 495°

$495 + 360 = 855°$ or $495 - 360 = 135°$

$135 - 360 = -225°$



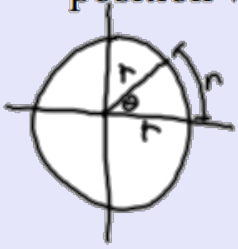
C. Radian Measure

So far, the only way you have measured angles is by degrees. You can measure an angle using a protractor to determine its measure.

Another way to measure an angle is to use **Radian Measure.** It is a way to measure an angle by the length of its arc. In theory, you could measure the angle with a tape measure (cm or in).

Definition of a Radian:

One radian is the measure of an angle is standard position whose terminal side intercepts an arc of length r.



So what does this mean?

So, some algebra to understand the concept of radians:

360° = Circumference of a Circle

$360^\circ = 2\pi r$

In a unit circle, we set the radius = 1
Therefore

360° = 2π radians
(This is a length)

$360 = 2\pi$

a. So what is 180° in radians? $180 = \frac{2\pi}{2} = \pi$

b. 90°? $\frac{\pi}{2}$

c. 45°? $\frac{\pi}{4}$

Here is the general way we can convert from radians to degrees and vice versa.

$$\frac{R}{D} = \frac{\pi}{180}$$

Ex. 3: Converting between radians and degrees.

$\frac{R}{D} = \frac{\pi}{180}$

a. Convert 110° to radians.

$\frac{x}{110} = \frac{\pi}{180}$

$\frac{180}{180}x = \frac{110\pi}{180}$

$x = \frac{11\pi}{18}$

b. Convert $-\frac{\pi}{9}$ radians to degrees.

$\frac{-\frac{\pi}{9}}{x} = \frac{\pi}{180}$

$-\frac{\pi}{9}(180) = x\pi$

$-\frac{20\pi}{\pi} = \frac{x\pi}{\pi}$

$x = -20°$

D. Let's put the whole lesson together!

You need to be able to:

- Draw an angle (in degrees or radians) in standard position.

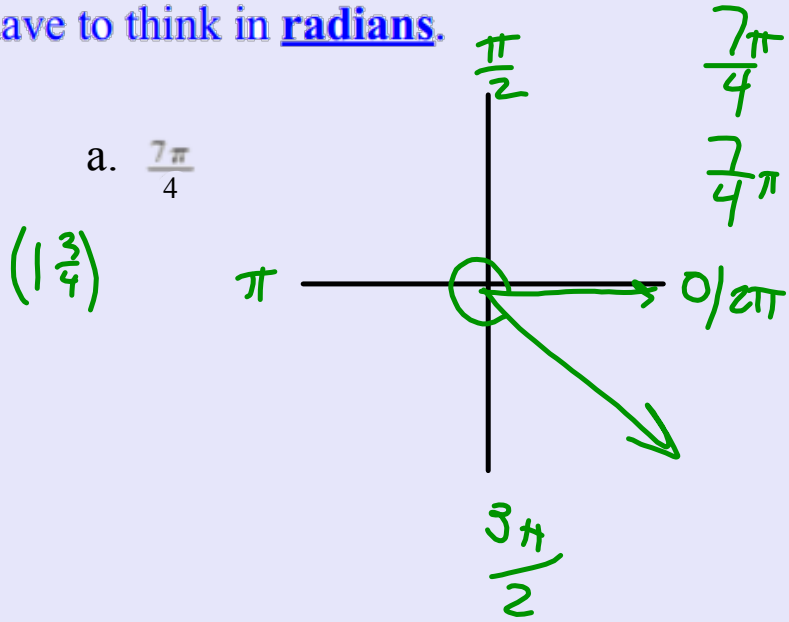
- Find positive and negative coterminal angles given an angle in either degree or radian measure

- Convert between degree and radian measure

Ex. 4. Draw an angle with the given measure in standard position. Then find one positive and one negative coterminal angle.

Tip: Work with me here with the fractions. You have to think in **radians**.

a. $\frac{7\pi}{4}$

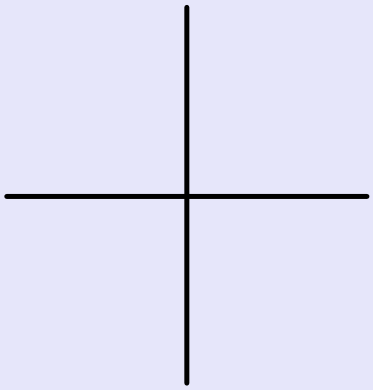


$\frac{7\pi}{4} + 2\pi = \frac{15\pi}{4}$

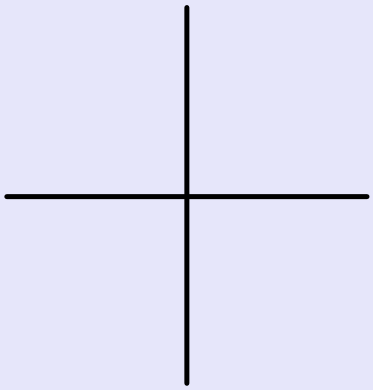
$\frac{7}{4}\pi - 2\pi = -\frac{\pi}{4}$

$(1\frac{3}{4})$

b. $-\frac{3\pi}{2}$



c. $\frac{7\pi}{8}$



E. Arc Length and Area of a Sector

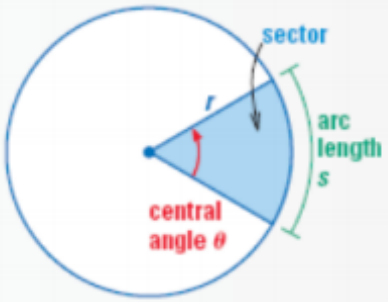
A **sector** is a region of a circle that is bounded by two radii and an arc of the circle. The **central angle** θ of a sector is the angle formed by the two radii. There are simple formulas for the arc length and area of a sector when the central angle is measured in radians.

ARC LENGTH AND AREA OF A SECTOR

The arc length s and area A of a sector with radius r and central angle θ (measured in radians) are as follows.

Arc length: $s = r\theta$

Area: $A = \frac{1}{2}r^2\theta$



Ex. 5 Find the arc length and area of a sector with a radius of 9 cm and a central angle of 60°.

Homework Assignment:

13.2

page 862-863 (1-37 odds)