12.5 Applications of Vectors

The main application of vectors is to problems that involve position, motion, and speed: the paths of two planes or ships over time, for example. The key to these applications is to remember the information that the vector equation gives us:

$$\mathbf{r} = \mathbf{a} + t\mathbf{b}$$

1) a is point on line. At the beginning of any event, t=0.

So at time $t = 0$, we get r=a+(0)b, or just r=a.

So, a represents the initial position of the person or object.

2) $\mathbf{b}$ is the velocity vector as a function of time. Every time-unit [second, minute, hour], the object moves $\mathbf{b}$ according to the x and y values.

The magnitude of b, $|\mathbf{b}|$ is the actual distance that the object moves every time-unit (second, minute, hour) in the direction of $\mathbf{b}$.

So $\mathbf{b}$ represents the movement of the object

And $|\mathbf{b}|$ represents the displacement / speed of the object.

3) A man's walk is shown by $\begin{bmatrix}5\\3\end{bmatrix}+t\begin{bmatrix}3\\4\end{bmatrix}$, where t is his time walking in seconds.

a) What is the man's initial position? $\begin{pmatrix}5\\3\end{pmatrix}$

b) Find the speed of the man. $\sqrt{3^2+4^2}=5$

c) What is the man's position after 10 seconds? How far has he walked?

position: $\begin{pmatrix}5\\3\end{pmatrix}+10\begin{pmatrix}3\\4\end{pmatrix}=\begin{pmatrix}35\\43\end{pmatrix}$ how far: $\sqrt{30^2+40^2}=50$

A woman's walk is shown by $\begin{bmatrix}-19\\27\end{bmatrix}+s\begin{bmatrix}4\\3\end{bmatrix}$, where s is her time walking in seconds.

d) Their paths will cross. Find the point where this happens.

e) Their paths will cross, but will they meet? Explain why or why not.

d. $\begin{pmatrix}5\\3\end{pmatrix}+t\begin{pmatrix}3\\4\end{pmatrix}=\begin{pmatrix}-19\\27\end{pmatrix}+s\begin{pmatrix}4\\3\end{pmatrix}$

$5+3t=-19+4s \quad (+24=-3t+4s)4$

$3+4t=27+3s \quad (-24=-4t+3s)3$

$t=24$

$s=24$

$\begin{pmatrix}5\\3\end{pmatrix}+24\begin{pmatrix}3\\4\end{pmatrix}=\begin{pmatrix}77\\99\end{pmatrix}$ ← paths cross here

e) Yes, because $t=24$ and $s=24$.

4) a) A toy boat is involved in a navigation exercise on a lake. It starts at the origin, O, and travels for 12 seconds with velocity $\begin{bmatrix} 5 \\ 6 \end{bmatrix}$ to point A. Find the coordinates of point A.

$A = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + 12\begin{pmatrix} 5 \\ 6 \end{pmatrix} = \begin{pmatrix} 60 \\ 72 \end{pmatrix}$

b) At A, the boat changes velocity to $\begin{bmatrix} 1 \\ -4 \end{bmatrix}$ and travels for 20 seconds to point B. Find the coordinates of point B.

$B = \begin{pmatrix} 60 \\ 72 \end{pmatrix} + 20\begin{pmatrix} 1 \\ -4 \end{pmatrix} = \begin{pmatrix} 80 \\ -8 \end{pmatrix}$

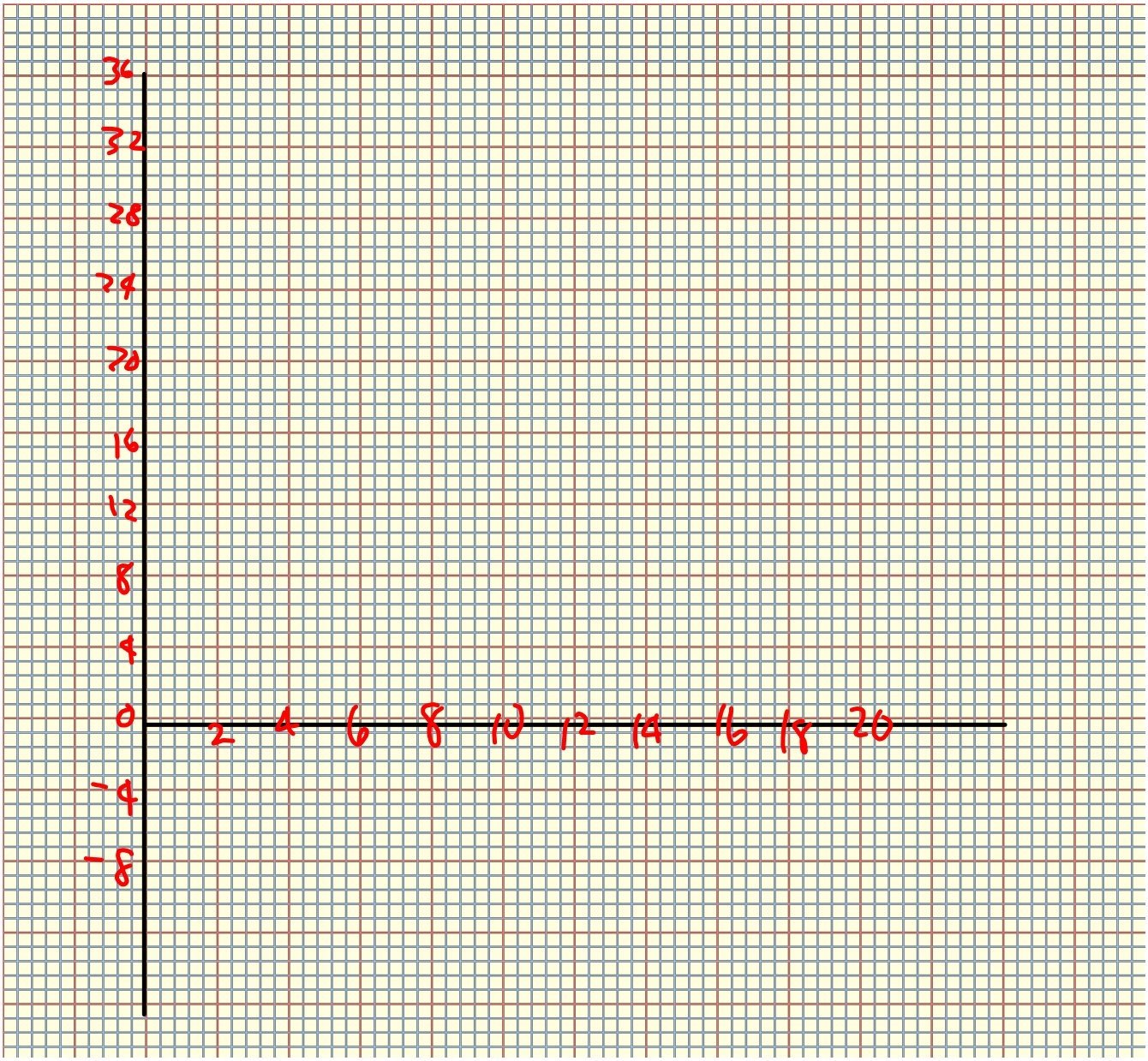
c) From B, the boat returns to O. Given that the journey from B to O takes 8 seconds, find in vector form an expression for its velocity on that final stage.

$\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 80 \\ -8 \end{pmatrix} + 8\begin{pmatrix} x \\ y \end{pmatrix}$

$0 = 80 + 8x \quad x = -10$

$0 = -8 + 8y \quad y = 1$

$\text{velocity} = \begin{pmatrix} -10 \\ 1 \end{pmatrix}$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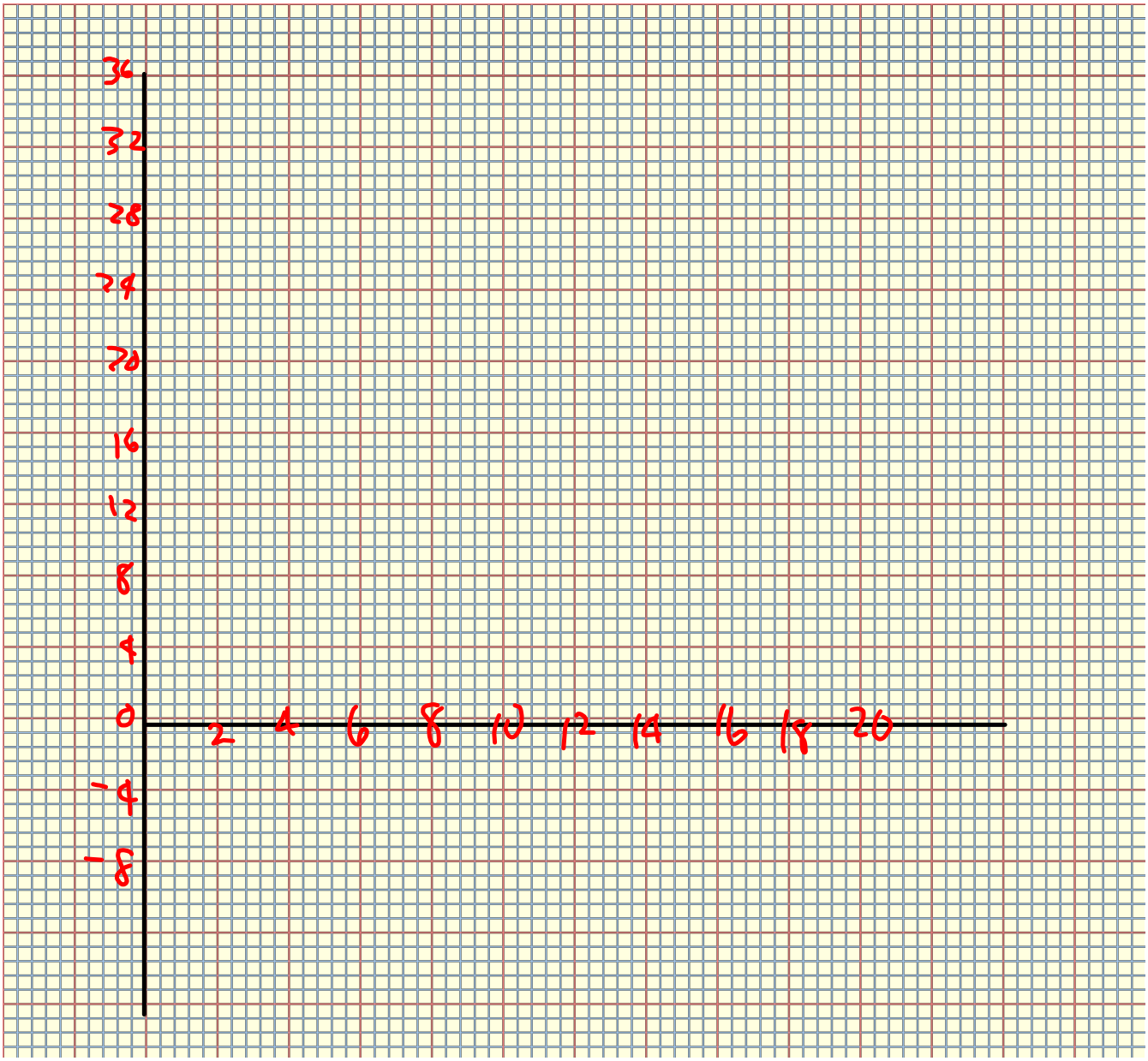
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