12.4 Vector Equation of a Line

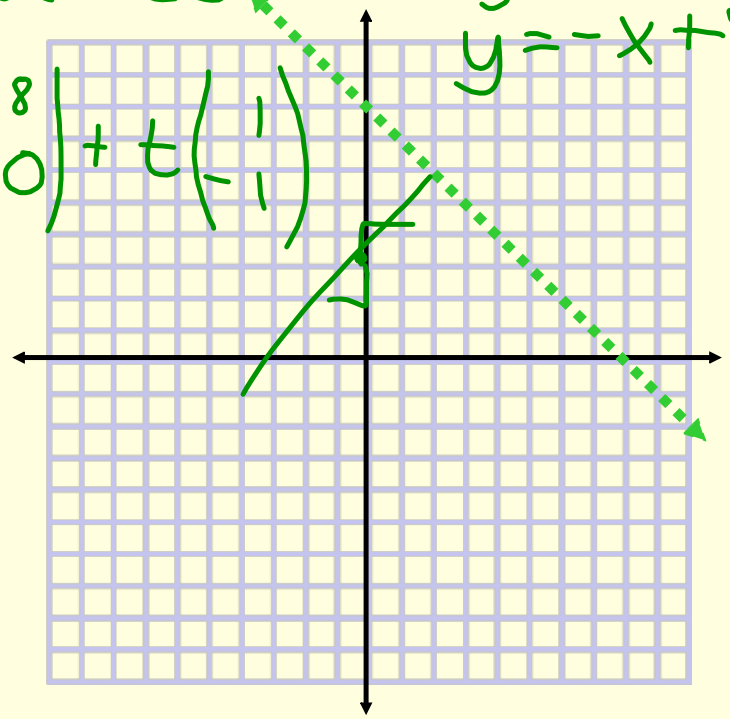
Vector Equation of a Line

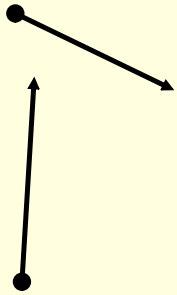
$r = a + tb$

$y = mx + b$

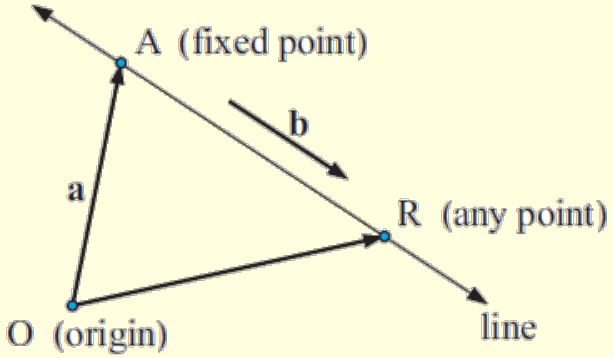
$y = -x + 8$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 0 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$



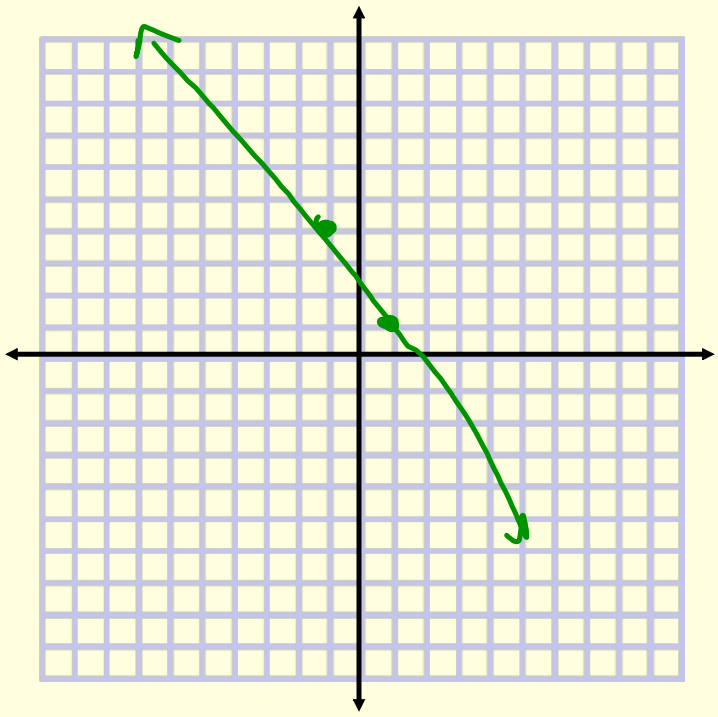


Vector Equation of a Line



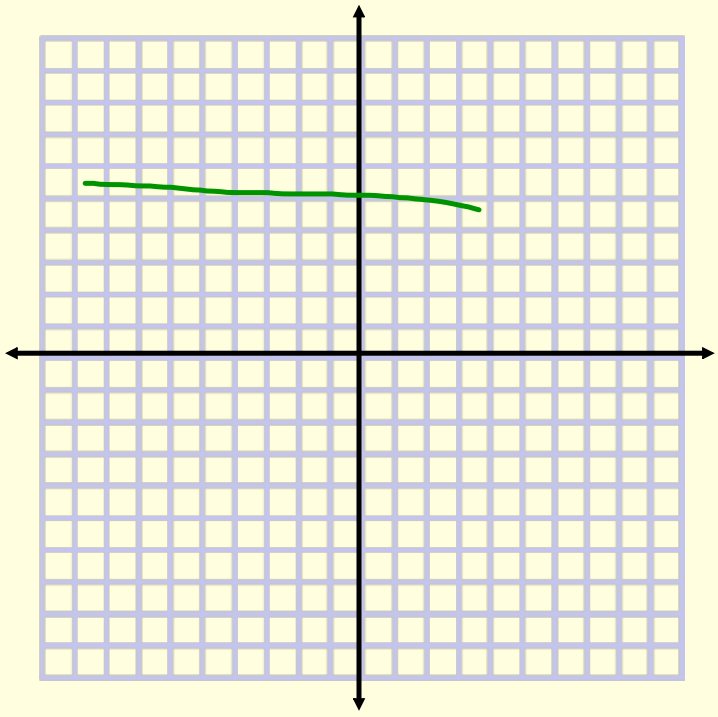
Vector Equation of a Line

$$\mathbf{r} = \begin{bmatrix} -1 \\ 4 \end{bmatrix} + t \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$



Vector Equation of a Line

$y = 5$



Vector Equation of a Line

1. Find the vector and Cartesian forms of a line passing through $A(1,3)$ and $B(2,5)$.

$m = \frac{2}{1} = 2$

$r = a + tb$

$r = \begin{pmatrix} 1 \\ 3 \end{pmatrix} + t\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$y - 3 = 2(x-1)$

$y = 2x - 2 + 3$

$y = 2x + 1$

Vector Equation of a Line

2. Find the vector and Cartesian forms of the line passing through $P(1,3,2)$ and $Q(0,-1,4)$. Does the point $R(-2,9,1)$ lie on the line PQ?

$$r = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} + t\begin{pmatrix} -1 \\ -4 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} -2 \\ 9 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} + t\begin{pmatrix} -1 \\ -4 \\ 2 \end{pmatrix}$$

$-2 = 1 - t$

$t = 3$

$9 = 3 - 4t$

$4t = -6$

$t = -\frac{6}{4}$

$1 = 2 + 2t$

$-1 = 2t$

$t = -\frac{1}{2}$

3. Find vector equations for a) $x = 2$ b) $2x - 5y = 3$

$r = a + tb$

$r = \begin{pmatrix} 2 \\ 0 \end{pmatrix} + t\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$y = mx + b$

r, b, t, a

4. Find a vector equation of the line perpendicular to $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and passing through the point $\begin{pmatrix} -1 \\ 6 \end{pmatrix}$.

$\begin{pmatrix} 3 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = 0$

$3x + 2y = 0$

$3(-2) + 2(3) = 0$

$r = \begin{pmatrix} -1 \\ 6 \end{pmatrix} + t\begin{pmatrix} -2 \\ 3 \end{pmatrix}$ $\quad b = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$

$\boldsymbol{v} \cdot \boldsymbol{w} = |\boldsymbol{v}||\boldsymbol{w}| \cos\theta$, where θ is the angle between $\boldsymbol{v}$ and $\boldsymbol{w}$

$\boldsymbol{v} \cdot \boldsymbol{w} = v_1 w_1 + v_2 w_2 + v_3 w_3$, where $\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$, $\boldsymbol{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$

$a \cdot b = \cos\theta\,|a||b|$

5. Find the angle between the lines $\mathbf{r}_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix} + \mu \begin{bmatrix} 3 \\ -4 \end{bmatrix}$ and $\mathbf{r}_2 = \begin{bmatrix} -2 \\ 3 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

$a \cdot b = -5$

$|a| = 5$

$|b| = \sqrt{5}$

$-5 = \cos\theta\,(5)(\sqrt{5})$

$\theta = 117^\circ$

6. Find the angle between the lines:

$$\mathbf{r}_1 = 2\mathbf{i} + \mathbf{j} + \lambda(\mathbf{i} + 3\mathbf{j})$$
$$\mathbf{r}_2 = 6\mathbf{i} - \mathbf{j} + \mu(\mathbf{i} - 4\mathbf{j})$$

$\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -4 \end{pmatrix}$

7. Find the coordinates of the point of intersection of these pairs of lines.

a) $l_1 : \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ -4 \end{pmatrix} + t\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $l_2 : \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 12 \\ -8 \end{pmatrix} + u\begin{pmatrix} -1 \\ 6 \end{pmatrix}$

b) $\mathbf{r}_1 = 6\mathbf{i} - 5\mathbf{j} + s(-2\mathbf{i} + \mathbf{j})$ $\mathbf{r}_2 = 5\mathbf{i} - 9\mathbf{j} + t(-\mathbf{i} + 2\mathbf{j})$

a) $\begin{pmatrix} 6 \\ -4 \end{pmatrix} + t\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 12 \\ -8 \end{pmatrix} + u\begin{pmatrix} -1 \\ 6 \end{pmatrix}$

$6 + t = 12 - u$

$-4 + 2t = -8 + 6u$

$-2($ $u + t = 6$)

$-6u + 2t = -4$

$-8u = 16$

$u = 2$

$6 + t = 10$

$t = 4$

$\begin{pmatrix} 4 \\ 2 \end{pmatrix}$

Find the point of intersection and the angle between the lines

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + s\begin{bmatrix} 3 \\ 1 \end{bmatrix} \qquad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ -4 \end{bmatrix} + t\begin{bmatrix} 1 \\ 2 \end{bmatrix}$$