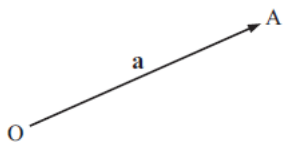
12.1, 12.2 Geometric Vectors and Basic Operations

Vector Vocabulary

scalar: size or magnitude only
vector: magnitude and direction

VECTOR NOTATION

Consider the vector from the origin O to the point A. We call this the **position vector** of point A.



- This **position vector** could be represented by
  $\overrightarrow{OA}$ or **a** or $\tilde{a}$ or $\overrightarrow{a}$
  (**a**: bold used in text books; $\tilde{a}$ or $\overrightarrow{a}$: used by students)
- The **magnitude** or **length** could be represented by
  $|\overrightarrow{OA}|$ or OA or $|\mathbf{a}|$ or $|\tilde{a}|$ or $|\overrightarrow{a}|$

For

we say that $\overrightarrow{AB}$ is the vector which **originates** at A and **terminates** at B,

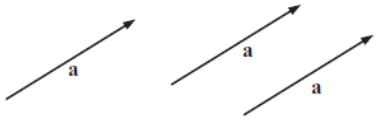
and that $\overrightarrow{AB}$ is the **position vector** of B relative to A.

GEOMETRIC VECTOR EQUALITY

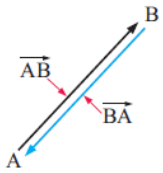
Two vectors are **equal** if they have the same magnitude and direction.

Equal vectors are **parallel** and in the same direction, and are **equal in length**. The arrows that represent them are translations of one another.

We can draw a vector with given magnitude and direction from *any* point, so we consider vectors to be **free**. They are sometimes referred to as **free vectors**.



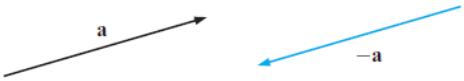
GEOMETRIC NEGATIVE VECTORS



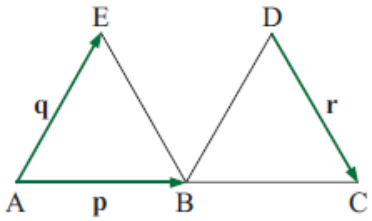
$\overrightarrow{AB}$ and $\overrightarrow{BA}$ have the same length, but they have opposite directions.

We say that $\overrightarrow{BA}$ is the negative of $\overrightarrow{AB}$ and write $\overrightarrow{BA} = -\overrightarrow{AB}$.

a and $-\mathbf{a}$ are parallel and equal in length, but opposite in direction.



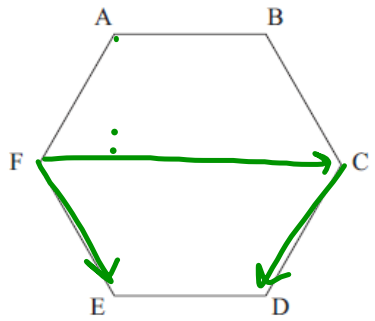
2 The figure alongside consists of two equilateral triangles. A, B, and C lie on a straight line.
$\overrightarrow{AB} = \mathbf{p}$, $\overrightarrow{AE} = \mathbf{q}$, and $\overrightarrow{DC} = \mathbf{r}$.
Which of the following statements are true?



a $\overrightarrow{EB} = \mathbf{r}$ T **b** $|\mathbf{p}| = |\mathbf{q}|$ T **c** $\overrightarrow{BC} = \mathbf{r}$ F

d $\overrightarrow{DB} = \mathbf{q}$ F **e** $\overrightarrow{ED} = \mathbf{p}$ T **f** $\mathbf{p} = \mathbf{q}$ F

3



ABCDEF is a regular hexagon.

a Write down the vector which:

i originates at B and terminates at C $\overrightarrow{BC}$

ii is equal to $\overrightarrow{AB}$. $\overrightarrow{ED}$

b Write down *all* vectors which:

i are the negative of $\overrightarrow{EF}$ $\overrightarrow{FE}$, $\overrightarrow{BC}$

ii have the same length as $\overrightarrow{ED}$.

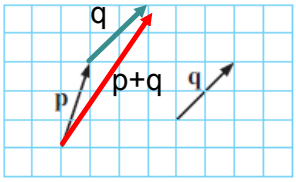
c Write down a vector which is parallel to $\overrightarrow{AB}$ and twice its length.

$\overrightarrow{FC}$

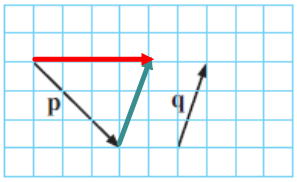
EXERCISE 14B.1

1 Use the given vectors $\mathbf{p}$ and $\mathbf{q}$ to construct $\mathbf{p} + \mathbf{q}$:

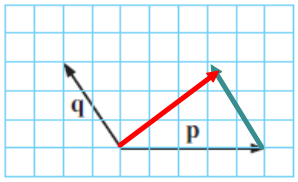
a



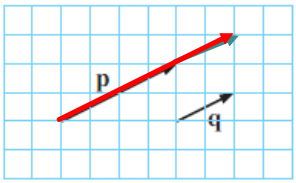
b



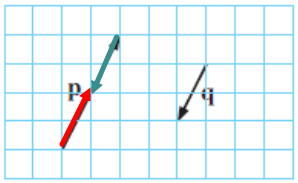
c



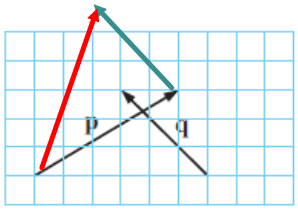
d



e



f



3 For points A, B, C, and D, simplify the following vector expressions:

a $\overrightarrow{AC} + \overrightarrow{CB}$ $\overrightarrow{AB}$

b $\overrightarrow{AD} - \overrightarrow{BD}$ $\overrightarrow{AB}$

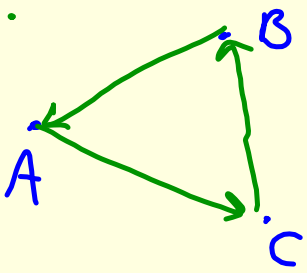
c $\overrightarrow{AC} + \overrightarrow{CA}$ 0

d $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}$ $\overrightarrow{AD}$

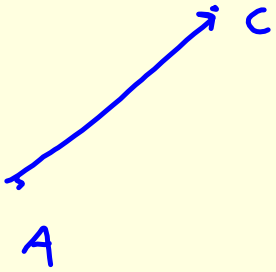
e $\overrightarrow{BA} - \overrightarrow{CA} + \overrightarrow{CB}$ $+\overrightarrow{AC} = 0$

f $\overrightarrow{AB} - \overrightarrow{CB} - \overrightarrow{DC}$ $\overrightarrow{AD}$

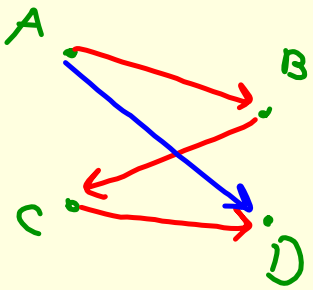
e.



C.

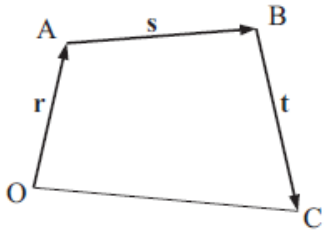


F.



2 **a** Find, in terms of **r**, **s**, and **t**:

i $\overrightarrow{OB}$ **ii** $\overrightarrow{CA}$ **iii** $\overrightarrow{OC}$



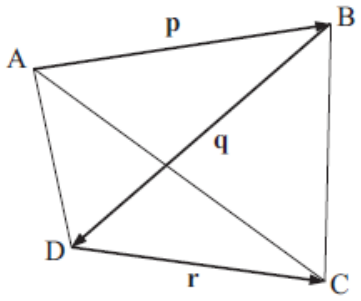
i. $r+s$

ii. $-t-s$

iii. $r+s+t$

b Find, in terms of **p**, **q**, and **r**:

i $\overrightarrow{AD}$ **ii** $\overrightarrow{BC}$ **iii** $\overrightarrow{AC}$

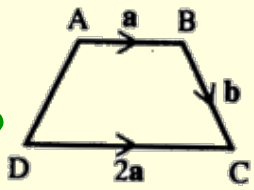


i. $p+q$

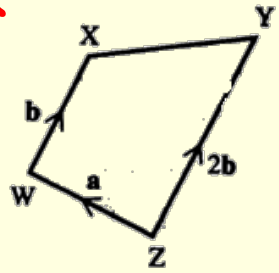
ii. $q+r$

iii. $p+q+r$

22. (a) $\overrightarrow{BA}$ $-a$
(b) $\overrightarrow{AC}$ $a+b$
(c) $\overrightarrow{DB}$ $2a-b$
(d) $\overrightarrow{AD}$ $a+b-2a$ $=b-a$



23. (a) $\overrightarrow{ZX}$
(b) $\overrightarrow{YW}$
(c) $\overrightarrow{XY}$
(d) $\overrightarrow{XZ}$



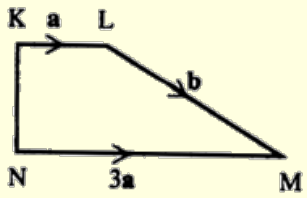
a) $a+b$

b) $-2b+a$

c) $-b-a+2b$
$=-a+b$ or $b-a$

d) $-b-a$

24. (a) $\overrightarrow{MK}$
(b) $\overrightarrow{NL}$
(c) $\overrightarrow{NK}$
(d) $\overrightarrow{KN}$



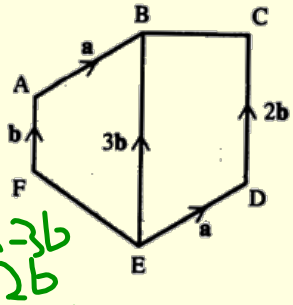
a) $-b-a$

b) $3a-b$

c) $3a-b-a=2a-b$

d) $a+b-3a=-2a+b$

25. (a) $\overrightarrow{FE}$
(b) $\overrightarrow{BC}$
(c) $\overrightarrow{FC}$
(d) $\overrightarrow{DA}$



a) $b+a-3b$
$=a-2b$

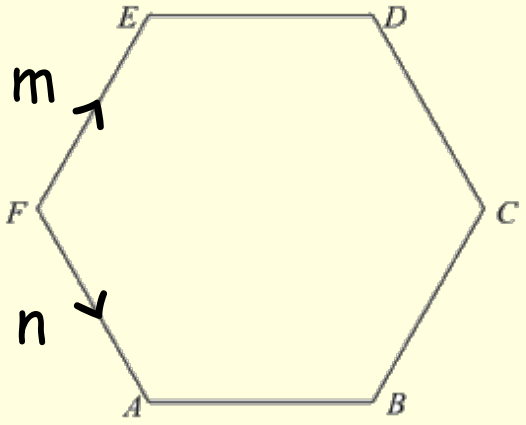
b) $-3b+a+2b$
$=a-b$

c) $b+a-3b+a+2b$
$=2a$

d) $-a+3b-a$
$=3b-2a$

ABCDEF is a regular hexagon with $\overrightarrow{FE}$ representing the vector m and $\overrightarrow{FA}$ representing the vector n.

Find the vector representing $\overrightarrow{FC}$.



$m = \overrightarrow{FE}$

$\overrightarrow{FA} = -m+n$

$n = \overrightarrow{FA}$